A simple model of working from home

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Abstract

The paper describes a simple model of working from home. The model extends the standard consumption vs unpaid hours decision faced by individuals to make labour a location-specific good. We drew three main insights from the model: (1) increased access to working from home increases labour supply as some time saved from commuting is diverted to working hours; (2) the commute is a major cost which is borne entirely by the individual who supplies labour — this cost drives much of the welfare improvements that occur when working from home is permitted and; (3) paying a different wage to office vs home-based labour yields an efficient outcome. However, when wages cannot vary by location, firms and workers will likely make adjustments over time to make the distribution of work more efficient; such as by investing in home-based work technologies, or by developing processes to make distributed work more productive.

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Introduction

The COVID-19 pandemic forced many workers and firms to experiment with working from home. While stay-at-home orders have eased across the developed world, the level of working from home is likely to remain much higher than pre-pandemic, as around 35-40 per cent of Australian jobs can be done remotely (Productivity Commission 2021, p.9). This research aims to shed light on how increased levels of working from home are likely to affect labour markets. It incorporates the concept that the productivity of individuals working in different locations may vary (Bloom *et al*. 2014), which could be related to management (Groen *et al*. 2018) and monitoring (Jensen *et al*. 2020) issues when workers are remote. It also takes into account varying preferences for working from home across employees (Barrero *et al*. 2021).

In response to the forced experiment of working from home caused by the COVID-19 pandemic, the Productivity Commission undertook a self-initiated research project to explore the possible economic effects of working from home. The paper considers how firms and workers will make decisions about the location of work when working from home is common. It also assesses the role of regulation in transitioning to working from home, including work place health and safety and workplace relations. The report also considers the likely effects on how cities are organised and the possible reallocation of economic activity. It concludes by discussing the potential effects of working from home on wellbeing and equity.

As part of the Productivity Commission's working from home report, we developed an illustrative working from home model that provides insights into how access to work from home might change agents' labour supply and demand decisions and what factors will influence the extent of those changes. The individual's (who supplies labour) preference for location is captured by entering the location of work directly into the utility function, which requires additional assumptions to ensure the utility function is realistic. We construct a simple hedonic labour model where labour units are broken into labour supplied/demanded from the home and labour supplied/demanded from the office. We assume that the firm values labour from each location based on its relative productivity, which can vary, while the individual has different preferences for each location and must also incur a time cost to work from the office (i.e. the commute). We consider equilibrium conditions when working from home is and is not permitted. A simulation exercise allows us to explore the implications of working from home on different groups for a range of parameter values.

We find that increased access to work from home would be expected to increase labour supply. The increase is likely to be larger for individuals with longer relative commutes, as the time saved from commuting can be distributed between work and non-work activities. The labour supply increase would also be expected to be largest among those people who have a stronger preference for working from home (for example, people with caring responsibilities, secondary earners in family households, and people with disabilities).

We find that the removal of the constraint which prohibits working from home (i.e. allowing working from home) unambiguously increases individual utility. This is primarily due to avoiding the cost of the commute, however preferences for the location of work also imply a gain for individuals with zero commute.

Finally, we find that if wages can adjust to reflect the location of work – that is, wages are allowed to differ according to whether work is done at home or in a central office location — then this allows for an optimal outcome for a wider range of individuals and firms. We discuss caveats to this finding, such as what parameter assumptions are required for this outcome and how these assumptions may, or may not, hold in the long run. We also discuss how working from home may develop over time if wages are not allowed to vary by location for a given individual.

The model

This illustrative working from home model provides insights into how access to work from home might change agents' labour supply and demand decisions and what factors will influence the extent of those changes. The model is necessarily an imperfect representation of a complex interaction between employers and employees, but yields useful insights. This paper briefly outlines the model, the model is then used as the basis of a simulation exercise which explores its implications.

These implications are summarised into three key insights which are addressed in the discussion section.

1. Increased access to work from home would be expected to increase labour supply. The increase is likely to be larger for individuals with longer relative commutes, as the time saved from commuting can be distributed between work and non work activities, based on the preferences of the individual. The labour supply increase would be expected to be largest among those people who have a stronger preference for work from home (for example, people with caring responsibilities, secondary earners in family households, and people with disabilities). If the productivity of home based work increases (relative to office based work), proportion of hours worked at home would also likely increase.
2. The commute is a time cost that could be better allocated to other activities like extra work, time with family, and caring or domestic tasks (Dockery and Bawa 2014; NSW IPC 2020). Because working from home allows individuals to avoid the commute it constitutes a weak Pareto improvement. Time otherwise spent commuting can be distributed between paid and unpaid activities, meaning that access to work from home unambiguously increases individual utility.

Because some of the reclaimed commuting time is devoted to work, the firm is at least as well off than before. This makes working from home beneficial for employers (assuming constant productivity across locations) and employees.

3. Flexible wages yield an efficient allocation of labour. If wages can adjust to reflect the location of work — that is, wages are allowed to differ according to whether work is done at home or in a common location — then this allows for an optimal outcome for a wider range of individuals and firms. However, when wages cannot adjust to reflect the location of work, parameters will likely evolve over time to improve the allocation of labour to the office and the home.

The model is a single period, two sided (individual and firm) problem that assumes the location of work is an important component of decisions made by individuals and firms. Work can be done in one of two locations: in the 'office', which represents typical centralised workplaces, or at 'home'. There are two agents in the model representing typical decision makers in the economy:

1. the individual (employee), who is assumed to maximise utility and supply labour
2. the firm (employer), who is assumed to maximise profit and demand labour.

The agents are a stylised representation of the aggregation of heterogenous employers and employees in the market. The model extends the standard consumption vs unpaid hours decision faced by individuals to include individual preferences for working from home ('flexibility') or in the office ('social interaction').

When individuals work from home they save the time that would otherwise be spent commuting. It is assumed that employers only have preferences for one work location over another if the location of work affects productivity.

Both agents are price takers, accepting the market wage for labour. As a starting point, it is assumed that different wages can be paid for each type of labour. However, the discussion section addresses what happens when wages for work performed at home and in the office are constrained to be equal.

The firm's problem

The employer is assumed to maximise profit (total revenue minus total cost). Output is a function of two kinds of productive hours of work, work supplied from the home and work supplied from the office. This makes the profit function of the firm:

$$\pi = p \cdot f(L_h, L_o) - w^h L_h - w^o L_o$$

Where:

1. π is profit
2. p is the unit price of the output good
3. L_h is hours worked at home per week
4. L_o is hours worked at the office per week
5. w^h is the wage paid to labour supplied from the home
6. w^o is the wage paid to labour supplied from the office.

If we specify a constant elasticity of substitution production function we get the following maximisation problem:

$$\max_{L_h,L_o} \pi = p\left(\beta_{L_h}{L_h}^{\rho} + \beta_{L_o}{L_o}^{\rho}\right)^{\frac{1}{\rho}} - w^h L_h - w^o L_o \; s.t. \;\; L_h \geq 0, \; L_o \geq 0$$

Where:

1. $\beta_{L_h} \in (0,1)$ is the output parameter for hours worked at home
2. $\beta_{L_o} \in (0,1)$ is the output parameter for hours worked at the office
3. $\beta_{L_h} + \beta_{L_0} = 1$
4. $\rho \in (-\infty, 1)$ is the substitution parameter, where $\sigma = \frac{1}{1-\rho}$ is the elasticity of substitution (that is, $\rho = \frac{\sigma-1}{\sigma}$)
5. The production function exhibits constant returns to scale.

Which yields the Lagrangian:

$$\mathcal{L} = p\left(\beta_{L_h}{L_h}^{\rho} + \beta_{L_o}{L_o}^{\rho}\right)^{\frac{1}{\rho}} - w^h L_h - w^o L_o + \lambda_{L_h} L_h + \lambda_{L_o} L_o$$

With the following first order conditions:

$$\frac{\partial \mathcal{L}}{\partial L_h} = p\beta_{L_h}{L_h}^{\rho-1}\left(\beta_{L_h}{L_h}^{\rho} + \beta_{L_o}{L_o}^{\rho}\right)^{\frac{1}{\rho}-1} - w^h + \lambda_{L_h} = 0 \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial L_o} = p\beta_{L_o}{L_o}^{\rho-1}\left(\beta_{L_h}{L_h}^{\rho} + \beta_{L_o}{L_o}^{\rho}\right)^{\frac{1}{\rho}-1} - w^o + \lambda_{L_o} = 0 \tag{2}$$

$$\lambda_{L_h} L_h = 0$$

$$\lambda_{L_o} L_o = 0$$

If we assume that the firm is using labour from both the home and the office, and combine equations (1) and (2):

$$\frac{\beta_{L_h}{L_h}^{\rho-1}}{\beta_{L_o}{L_o}^{\rho-1}} = \frac{w^h}{w^o}$$

$$\frac{L_h}{L_o} = \left(\frac{\beta_{L_o} w^h}{\beta_{L_h} w^o}\right)^{\frac{1}{\rho-1}} \tag{a}$$

This implies that the firm's labour demand will be determined by the wage ratio, the relative productivity of home labour to office labour, and the substitutability of home and office labour.

If wages are fixed to be equal regardless of where work is performed, then the wage ratio will be 1. In this case, the distribution of labour across the two locations will be determined only by the relative marginal profit from using employees in the two locations (as determined by β_{L_o} and β_{L_h}) and how readily the firm can substitute between them.

The individual's problem

The individual enjoys consumption, unpaid activities, and has a preference affecting where they want to work. The preference for the location of work simply means that hours worked in the home are valued differently to hours worked in the office. Preferences for a particular location of work reflects the value of social interaction when working at the office and the value of flexibility when working at home. The individual then faces the following utility function:

$$U(C, H, L_h, L_o)$$

Where:

1. C is consumption of the individual, which includes the benefits of savings
2. H is the unpaid hours of the individual — these hours can be used for leisure, house work, extra sleep etc.
3. L_h is the hours worked at home, here it reflects a direct preference for the location of work to be the home. We assume that $\frac{\partial U}{\partial L_h} > 0$, $\frac{\partial^2 U}{\partial L_h^2} < 0$
4. L_o is the hours worked in the office, here it reflects a direct preference for the location of work to be the office. We assume that $\frac{\partial U}{\partial L_o} > 0$, $\frac{\partial^2 U}{\partial L_o^2} < 0$.

The individual's constraints are time and budget. The individual's time is strictly bounded, with time divided up between paid activities (work) and unpaid activities. The individual's consumption is bounded by their earnings from labour (i.e. their budget).

This set up breaks from convention as labour hours enter directly into the utility function instead of only through consumption and unpaid time. This implies that there is a positive marginal utility of labour in this model. The aim of this feature is to capture the benefits of the location of work separate to the typical trade off between consumption and unpaid time.

Wanting more work in the office (captured by the fourth term in the utility function) is intended to represent people who get satisfaction from socialising with colleagues. Wanting more work at home (captured by the third term in the utility function) represents people who enjoy working in familiar surroundings, comfortable clothes, and being able to do home activities in between work tasks.

We impose the restriction that $\frac{\partial U}{\partial H} > \frac{\partial U}{\partial L_o}$ and $\frac{\partial U}{\partial H} > \frac{\partial U}{\partial L_h}$ for all values of L_o and L_h below $\bar{T}$, so that the preference for a certain type of work is not so strong that the worker would agree to work for free (or a negative wage).

The unpaid hours constraint is:

$$H = \bar{T} - L_h - (1+t)L_o$$

Where:

1. $\bar{T}$ is the total work week time endowment
2. t is the fixed length of commute expressed as a fraction of hours spent at the office.

For the individual income constraint, we assume no savings, so income (Y) is equal to consumption (C):

$$Y = C = w^h L_h + w^o L_o$$

This gives the following maximisation problem:

$$\max_{C,H,L_h,L_o} U(C,H,L_h,L_o)$$

$$s.t. \quad H + L_h + L_o + \mathrm{t}L_o \leq \bar{T}$$

$$\mathrm{C} \leq w^h L_h + w^o L_o$$

$$L_h \geq 0,\ L_o \geq 0$$

Substituting the constraints into the utility function yields:

$$\mathcal{L} = U(w^h L_h + w^o L_o\ , \bar{T} - L_h - (1+t)L_o,\ L_h,\ L_o) + \lambda_{L_h} L_h + \lambda_{L_o} L_o$$

We assume that an interior solution exists in which the individual wants to work a positive number of hours, and have some positive number of unpaid hours, which holds if $\lim_{C\to 0} \frac{\partial U}{\partial C} = \infty$ and $\lim_{H\to 0} \frac{\partial U}{\partial H} = \infty$.

This formulation allows for the possibility that the employee could choose to work almost all of their hours at the office, or almost all of their hours at home. If a positive number of hours are worked at home, then the parameter $\lambda_{L_h} = 0$, and if a positive number of hours are worked at the office, then the parameter $\lambda_{L_o} = 0$.

Taking first order conditions (assuming an interior solution) with respect to L_h and L_o yields:

$$\frac{\partial \mathcal{L}}{\partial L_h} = \frac{\partial U}{\partial C} w^h - \frac{\partial U}{\partial H} + \frac{\partial U}{\partial L_h} + \lambda_{L_h} = 0 \tag{3}$$

$$\frac{\partial \mathcal{L}}{\partial L_o} = \frac{\partial U}{\partial C} w^o - \frac{\partial U}{\partial H} \cdot (1+t) + \frac{\partial U}{\partial L_o} + \lambda_{L_o} = 0 \tag{4}$$

$$\lambda_{L_h} L_h = 0, \qquad L_h \geq 0$$

$$\lambda_{L_o} L_o = 0, \qquad L_o \geq 0$$

Equations (3) and (4) imply that as the marginal utility of consumption $\left(\frac{\partial U}{\partial C}\right)$ increases, the labour supplied from both home and office will increase, assuming L_o>0

and L_h>0. Conversely, as the marginal utility of unpaid hours $\left(\frac{\partial U}{\partial H}\right)$ increases, labour supplied from the home and office will decrease.

Equations (3) and (4) also show that the longer it takes for employees to commute to and from the office — that is, as t increases — the larger is $\frac{\partial U}{\partial L_o}$ (holding other things constant) and the smaller is $\frac{\partial U}{\partial L_h}$, meaning more time will be spent at home.[1] Even if the wages for home and office work are identical, there could be an interior solution, depending on the values of social interaction and avoiding the commute.

If we specify a constant elasticity of substitution utility function (without substituting the constraints into the utility function), the maximisation problem becomes:

$$\mathcal{L} = \left(\alpha_C C^r + \alpha_H H^r + \alpha_{L_h} {L_h}^r + \alpha_{L_o} {L_o}^r\right)^{\frac{1}{r}} + \lambda_H(-H - L_h - L_o - \mathrm{t}L_o + \bar{T}) + \lambda_C(w^h L_h + w^o L_o - \mathrm{C}) + \lambda_{L_h} L_h + \lambda_{L_o} L_o$$

Where:

1. $\alpha_C \in (0,1)$ is the preference parameter for consumption
2. $\alpha_H \in (0,1)$ is the preference parameter for unpaid hours
3. $\alpha_{L_h} \in (0,1$ is the preference parameter for hours worked at home
4. $\alpha_{L_o} \in (0,1)$ is the preference parameter for hours worked at the office
5. $\alpha_C + \alpha_H + \alpha_{L_h} + \alpha_{L_o} = 1$
6. $r \in (-\infty, 1)$ is the substitution parameter, where $s = \frac{1}{1-r}$ is the elasticity of substitution
7. The utility function exhibits constant returns to scale.

Which yields the following first order conditions:

$$\frac{\partial \mathcal{L}}{\partial C} = \alpha_C C^{r-1}\left(\alpha_C C^r + \alpha_H H^r + \alpha_{L_h} {L_h}^r + \alpha_{L_o} {L_o}^r\right)^{\frac{1}{r}-1} - \lambda_C = 0 \quad (5)$$

$$\frac{\partial \mathcal{L}}{\partial H} = \alpha_H H^{r-1}\left(\alpha_C C^r + \alpha_H H^r + \alpha_{L_h} {L_h}^r + \alpha_{L_o} {L_o}^r\right)^{\frac{1}{r}-1} - \lambda_H = 0 \quad (6)$$

$$\frac{\partial \mathcal{L}}{\partial L_h} = \alpha_{L_h} L_h^{r-1}\left(\alpha_C C^r + \alpha_H H^r + \alpha_{L_h} {L_h}^r + \alpha_{L_o} {L_o}^r\right)^{\frac{1}{r}-1} - \lambda_H + \lambda_C w^h + \lambda_{L_h} = 0 \quad (7)$$

$$\frac{\partial \mathcal{L}}{\partial L_o} = \alpha_{L_o} L_o^{r-1}\left(\alpha_C C^r + \alpha_H H^r + \alpha_{L_h} {L_h}^r + \alpha_{L_o} {L_o}^r\right)^{\frac{1}{r}-1} - \lambda_H(1+t) + \lambda_C w^o + \lambda_{L_o} = 0 \quad (8)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_H} = H + L_h + L_o + \mathrm{t}L_o - \bar{T} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_C} = w^h L_h + w^o L_o - \mathrm{C} = 0$$

$$\lambda_H \geq 0, \quad \lambda_C \geq 0, \quad \lambda_{L_h} \geq 0, \quad \lambda_{L_o} \geq 0$$

$$\lambda_H(H + L_h + L_o + \mathrm{t}L_o - \bar{T}) = 0, \quad \lambda_C(w^h L_h + w^o L_o - \mathrm{C}) = 0,$$

$$\lambda_{L_h} L_h = 0, \quad \lambda_{L_o} L_o = 0$$

1 Remembering that a low marginal utility of labour reflects a high level of labour.

The section below considers these first order conditions when the individual can and cannot work from home. When the individual does not have the option to work from home, the maximisation problem for the firm and individual would need to be modified so that the firm and individual do not have the option to choose L_h. These maximisation problems have not been written out for the sake of brevity.

Equilibrium conditions when working from home is permitted ('work from home')

The first order conditions of the individual's maximisation problem help us understand how parameters of the model affect equilibrium levels of paid and unpaid hours when individuals can and cannot work from home. Focusing on the range of values for which workers want to work a positive amount from home and from the office ($\lambda_{L_h} = 0,\ \lambda_{L_o} = 0$), we use (5) and (6) to substitute λ_H and λ_C out of equations (7) and (8):

$$\alpha_{L_h} L_h^{r-1} - \alpha_H H^{r-1} + w^h \alpha_C C^{r-1} = 0 \tag{7'}$$

$$\alpha_{L_o} L_o^{r-1} - (1+t)\alpha_H H^{r-1} + w^o \alpha_C C^{r-1} = 0 \tag{8'}$$

Combining (7') and (8') to eliminate H yields:

$$\alpha_{L_o} L_o^{r-1} - (1+t)\alpha_{L_h} L_h^{r-1} + w^o \alpha_C C^{r-1} - (1+t) w^h \alpha_C C^{r-1} = 0$$
$$\alpha_{L_o} L_o^{r-1} - (1+t)\alpha_{L_h} L_h^{r-1} = \left((1+t)w^h - w^o\right)\alpha_C C^{r-1}$$

Replacing C with its value in terms of L_o and L_h from the income equation yields:

$$\alpha_{L_o} L_o^{r-1} - (1+t)\alpha_{L_h} L_h^{r-1} = \left((1+t)w^h - w^o\right)\alpha_C (w^h L_h + w^o L_o)^{r-1}$$
$$\alpha_{L_o} - (1+t)\alpha_{L_h} \left(\frac{L_h}{L_o}\right)^{r-1} = \left((1+t)w^h - w^o\right)\alpha_C \left(w^h \frac{L_h}{L_o} + w^o\right)^{r-1} \tag{b}$$

This (b) function implicitly defines the $\frac{L_h}{L_o}$ ratio. In particular:

- if $w^o = (1+t)w^h$ (i.e. the office wage and the home wage are identical, after adjusting for the cost of travel), then $L_h = L_o \left(\frac{(1+t)\alpha_{L_h}}{\alpha_{L_o}}\right)^{\frac{1}{1-r}}$ and the utility of consumption α_C does not affect the relative proportions of home and office work. Increasing the fixed commute length will shift work towards the home.
- $w^o = (1+t)w^h$ (i.e. the office wage is higher), then $L_h < L_o \left(\frac{(1+t)\alpha_{L_h}}{\alpha_{L_o}}\right)^{\frac{1}{1-r}}$ and L_h is a smaller share as the utility of consumption α_C is larger. L_h becomes a smaller share of labour as the utility of consumption increases because L_o will grow faster than L_h (because of the relatively higher returns to L_o).
- $w^o = (1+t)w^h$ (i.e. the office wage is lower), then $L_h > L_o \left(\frac{(1+t)\alpha_{L_h}}{\alpha_{L_o}}\right)^{\frac{1}{1-r}}$ and L_h is larger as the utility of consumption α_C is larger.
- Intuitively, $\frac{L_h}{L_o}$ increases with α_{L_h} and decreases with α_{L_o}.

Equilibrium conditions when working from home is not permitted ('no work from home')

Before 2020, most offices did not allow many workers the option of regularly working from home. In that state of the world, which we will describe as 'no work from home', if $L_h=0$ and $L_o>0$, the optimisation conditions are:

$$\frac{\partial \mathcal{L}}{\partial C} = \alpha_C C^{r-1}\left(\alpha_C C^r + \alpha_H H^r + \alpha_{L_o} {L_o}^r\right)^{\frac{1}{r}-1} - \lambda_C = 0 \tag{9}$$

$$\frac{\partial \mathcal{L}}{\partial H} = \alpha_H H^{r-1}\left(\alpha_C C^r + \alpha_H H^r + \alpha_{L_o} {L_o}^r\right)^{\frac{1}{r}-1} - \lambda_H = 0 \tag{10}$$

$$\frac{\partial \mathcal{L}}{\partial L_o} = \alpha_{L_o} L_o^{r-1}\left(\alpha_C C^r + \alpha_H H^r + \alpha_{L_o} {L_o}^r\right)^{\frac{1}{r}-1} - \lambda_H(1+t) + \lambda_C w^o = 0 \tag{11}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_H} = H + L_o + \mathrm{t}L_o - \bar{T} = 0 \tag{12}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_C} = w^o L_o - \mathrm{C} = 0 \tag{13}$$

Using (9) and (10) to substitute λ_H and λ_C out of equation (11) yields:

$$\alpha_{L_o} L_o^{r-1} - (1+t)\alpha_H H^{r-1} + w^o \alpha_C C^{r-1} = 0$$

Using (12) and (13) to substitute out H and C and solve for L_o yields:

$$\alpha_{L_o} L_o^{r-1} - (1+t)\alpha_H (\bar{T} - (1+t)L_o)^{r-1} + {w^o}^r \alpha_C {L_o}^{r-1} = 0$$

Re arranging:

$$\left(\frac{(1+t)\alpha_H}{\alpha_{L_o} + {w^o}^r \alpha_C}\right)^{\frac{1}{1-r}} + (1+t) = \frac{\bar{T}}{L_o}$$

Solving for L_o yields:

$$L_o = \frac{\bar{T}}{\left(\frac{(1+t)\alpha_H}{\alpha_{L_o} + {w^o}^r \alpha_C}\right)^{\frac{1}{1-r}} + (1+t)}$$

Intuitively, labour supply to the office is high when α_C is high, when α_{L_o} is high, when the wage is high, when α_H is low, or t is low.

Simulation setup

To explore the implications of a range of equilibrium outcomes given specific parameter values of the model, simulations were run using the General Algebraic Modelling System (GAMS) software. GAMS was used to program and solve the optimisation problem for a range of parameter values.

The simulations were done using the first order conditions in equations (1), (2), (5), (6), (7), (8) and their respective constraints (unless otherwise stated).

The default set of parameters used in simulations are described in table 1. A slightly higher preference for unpaid hours than consumption is chosen because of the specification of the utility function. The direct entry of labour into the utility function will otherwise skew preferences towards more work creating unintuitive patterns of substitution.

The fixed commute length of $t = 0.125$ was chosen as it represents a 1 hour total commute for an 8 hour work day, which approximates the average commute of full-time workers in Australian major cities in 2019 (67 minutes) (Productivity Commission 2021, p.3).

Labour was set to be equally productive in both locations by default as evidence on this is mixed. Survey data has found that 75 per cent of Australians believe they are as or more productive at home than in the office (Beck and Hensher 2021). Some evidence has shown that workers can be more productive at home if they are allowed to sort to their preferred location (Bloom *et al*. 2015), while other evidence collected during the pandemic found working from home decreased productivity (Gibbs, Mengel and Siemroth 2021).

Substitution parameters were chosen so that the degree of substitutability between inputs is higher than the degree of complementarity. Total time endowment was set to $\bar{T} = 80$ as this represents the total number of hours in the 5 work week days (assuming individuals sleep for 8 hours per night).

Table 1: Default parameter settings for simulations

α_C	0.47
α_H	0.49
α_{L_h}	0.02
α_{L_o}	0.02
t	0.125
β_{L_h}	0.5
β_{L_o}	0.5
$s\ (individual)$	2
$\sigma\ (firm)$	2
$\bar{T}$	80

Discussion

Working from home leads to increased labour supply

A number of factors influence how much people would choose to work at home or in the office if the choice was entirely theirs. Broadly, these can be summarised as:

- the trade-off between the time people have for themselves, family and friends
- the consumption they get from the income earned through work
- any additional wellbeing benefits (such as social interaction) or costs (such as stress or effort) associated with that work.

Ultimately, individuals must decide how to allocate their finite weekly hours between different activities.

When work is constrained to the office, people's labour supply decisions are more straightforward but less flexible. For those who would work, travel time is a necessary cost of getting to the workplace and an unavoidable ingredient to obtaining income for consumption. But the commute is time spent neither working nor on leisure/home production, and is lost.

Figure 1 shows a simulation where only the individual's maximisation problem was considered. It shows the number of hours the individual is willing to work for a given wage and set of preference parameters when the commute length is varied. The parameters are set as shown in table 1 and both wages set to 1 – that is, $w^o = w^h = 1$. Then the value for t is varied, indicating how a change in the individual's fixed commute length will, holding all other variables constant, affect their labour supply.

The dark blue line shows the outcome when working from home is not available. This is imposed using an additional constraint (not included in the first order conditions and formulation above, for simplicity). That additional constraint will have its own Lagrange multiplier. The individual would like to choose to work from home, but the additional constraint prevents it. The light blue line shows when this constraint is removed and working from home is made possible.

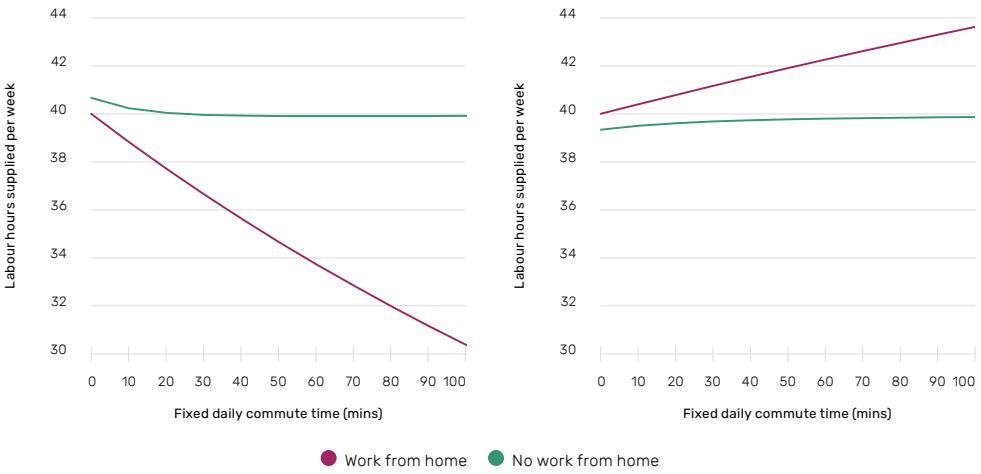
The small difference between the two curves at zero commute is due to the small degree of complementarity between home and office work in the utility function. Intuitively, the complementarity between home and office labour implies that an individual would get a higher payoff from devoting 15 hours to home labour and 15 hours to office labour than they would get from devoting all 30 hours to either one. This complementarity could be thought of as reflecting a preference for hybrid working arrangements.

As the fixed length of the commute increases, it becomes more costly to go into the office. Individuals who can work from home substitute more labour hours into home based work and their overall labour supply changes very little. Due to the fact that this is a single agent model, this fixed length of the commute is best represented as the distance lived from the office. However, in a dynamic multi-agent system it could also

represent a variable that is dependent on the actions of others, such as the level of traffic congestion. Increased working from home is unlikely to decrease congestion without the support of other policies such as road pricing (Productivity Commission 2021, p.55). Even with more people working from home, congestion may worsen if people who used public transport prior to the pandemic begin driving.

However, individuals without the option to work from home substitute work hours for unpaid hours and overall labour supply decreases. Because wages are fixed in this scenario, the utility of the individual who cannot work from home decreases as the commute gets longer, whereas the utility for the home based worker does not change.

Figure 1. Allowing home-based work increases the supply of labour — especially with longer commute times[a]



a. The values on the horizontal axis represent $t=\{0,0.02,0.04,...,0.18\}$, noting that t is the fixed length of the commute per hour worked, but is only incurred when the person works in the office. So if we assume that a person who works in the office does so for 8 hours, then a commute length of $t=0.02$ represents a fixed commute length of 9.6 minutes (rounded to 10).

Labour supply increases are largest for those with the strongest attachment to the home

People's preferences are an important factor influencing their attitude toward work. In this model the individual trades off between consumption and unpaid hours as well as between office based and home based work.

In determining labour supply, people consider the importance of time spent not working compared to consumption and paid work. These preferences are reflected in their utiltiy weights on unpaid hours and consumption, α_H and α_C, respectively. For

example, people with a relative preference for more non work time ($\alpha_H > \alpha_C$), even if it means lower income, might have carer responsibilities.

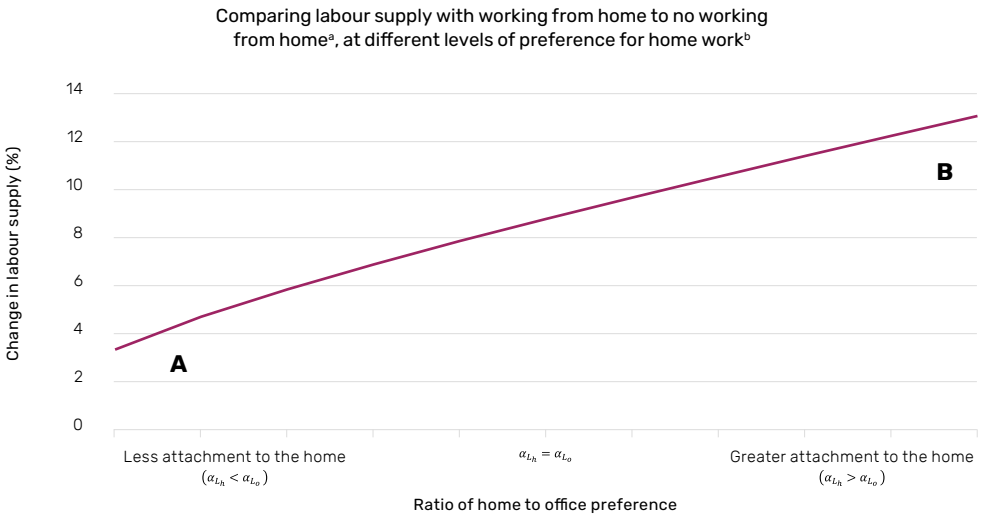
At the same time, the individual will consider how much they want to work at home relative to the office. People with a higher utility weight for home labour than office labour ($\alpha_{L_h} > \alpha_{L_o}$), are people who would prefer more of their work time to be at home. This could include people with disabilities that make workplace attendance challenging, or people who appreciate the flexibility to substitute between unpaid home production and paid work. The distribution of which people will have these different preferences will be determined by those who are in jobs that can be done form home. Working from home is particularly suited to office based workers such as managers, professionals and clerical and administrative workers, where workers use computers, interact less with the public, do not perform outdoor work or physical activity, and do not work with immovable structures, materials or equipment. This potential to work from home is associated with higher levels of education and higher incomes, and full time jobs.

Figure 2 is similar to figure 1 in that it shows only the individual's optimal conditions without considering the demand for labour (wages are once again fixed to be equal to 1). It shows how much an individual's labour supply increases when work from home is made available for people with varying relative preferences for home labour vs office labour.

People who prefer office based work (or actively dislike work from home) will not substantially change their work location decision when home based work is made available (point A on figure 2). This means that the option to work from home will not increase their labour supply by very much for a given wage. This also limits their welfare gain as they do not save much time from avoiding the commute.

However, as people's relative preference for home based work increases, the effect of being able to work from home increases their labour supply response. They increase their number of hours worked by a greater percentage (as shown in figure 2 comparing the curve at point A to point B). This also means people with these preferences receive the greatest increase in their welfare.

Figure 2. Those with the strongest attachment to the home will have the biggest labour supply response



a. This measures the difference in total labour supply when there is no working from home to when there is working from home.

b. This measures the relative direct preference of home to office work, i.e. $\alpha_{L_h}/\alpha_{L_o}$. Specifically, the horizontal axis shows the relative preference for location of labour, where 'less attachment to the home' indicates $\alpha_{L_h} < \alpha_{L_o}$ and strong attachment to the home indicates $\alpha_{L_h} < \alpha_{L_o}$.

This higher level of labour supply for people with a larger $\frac{\alpha_{L_h}}{\alpha_{L_o}}$ is the result of a greater desire to work at home, rather than a desire to work more to get more consumption.

Improved productivity of home based work will increase demand for home based labour

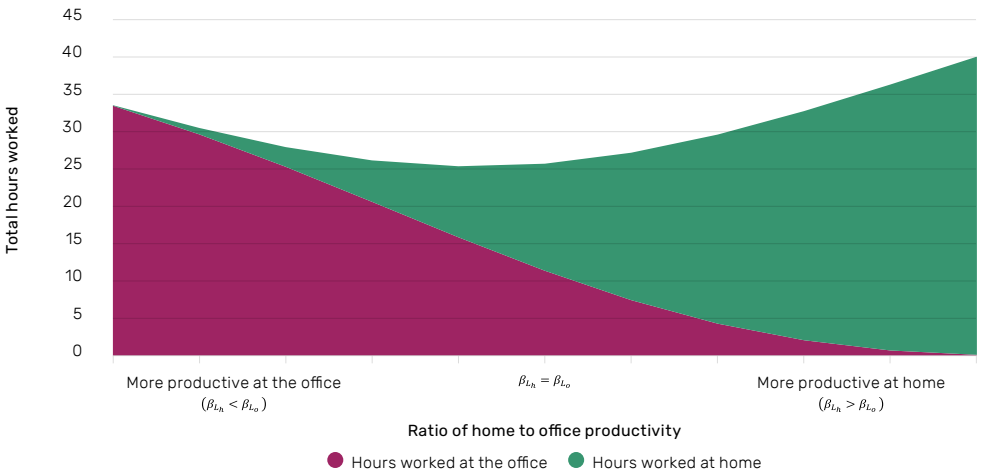
If we consider the firm and the individual together, then the total amount of labour traded in the market will also be determined by the marginal productivity of labour. As the relative productivity of home based work increases, employers will have a greater willingness to make use of it, with flow on implications for individuals.

Figure 3 shows how changes in the productivity of working from home relative to working in the office relates to the total number of hours worked. It shows that, if firms observe increased productivity in home based work, they will be more willing to scale up the share of work hours that are from the home, even when home based work remains less productive than office based work.

The largest amount of labour is traded in the market when home based production is much more productive than office based production. However, when the

firm's office productivity is similar to the productivity of home based labour there is a lower level of labour traded than when the firm specialises in either home or office-based technology. This is because the relative increase in wages paid to home labour does not entice the individual to work more at home than they work less in the office. Instead, they substitute much of their saved time to unpaid hours.

Figure 3. Firms will adopt more home-based work as its relative productivity improves[a]



a. The horizontal axis shows the relative weights on home and office labour in the production function.

In the figure above, the values on the horizontal axis indicate various ratios of home to office productivity (i.e. $\frac{\beta_{L_h}}{\beta_{L_o}}$). How this ratio affects the firm's labour demand can been seen in equation (a), reproduced below.

$$\frac{L_h}{L_o} = \left(\frac{\beta_{L_o} w^h}{\beta_{L_h} w^o}\right)^{\frac{1}{\rho-1}}$$

Remembering again that $\rho \in (-\infty, 1)$, this shows that, holding all other variables constant, an increase in the ratio of home to office productivity $\left(\frac{\beta_{L_h}}{\beta_{L_o}}\right)$ will lead to an increase in the ratio of home based to office based work. When home based work is substantially more productive, overall labour traded is higher. This is because the commute is being avoided by the individual and they have more time overall.

There are a number of factors which could influence the equilibrium outcome between employees and employers, which are not captured in the stylised model framework used to produce figure 3. For example, the unconstrained version of the model

implies that the equilibrium ratio of office to home labour hours will be jointly determined by the preferences of the individual and the production and cost functions of the firm. In reality, equilibrium working arrangements are more likely to be determined by relative bargaining power – often with the firm setting a 'work from home' policy for employees to follow. Over time, employees who highly value the ability to work from home may change jobs or accept lower wages in order to continue working from home. Job switching and negotiation on wages provides firms with information about what attracts (desirable) workers. In this way, switching and negotiation creates a process of experimentation in which firms try different arrangements, observe outcomes, relinquish unsuitable arrangements and maintain those that yield desirable results.

The commute can be seen as a cost

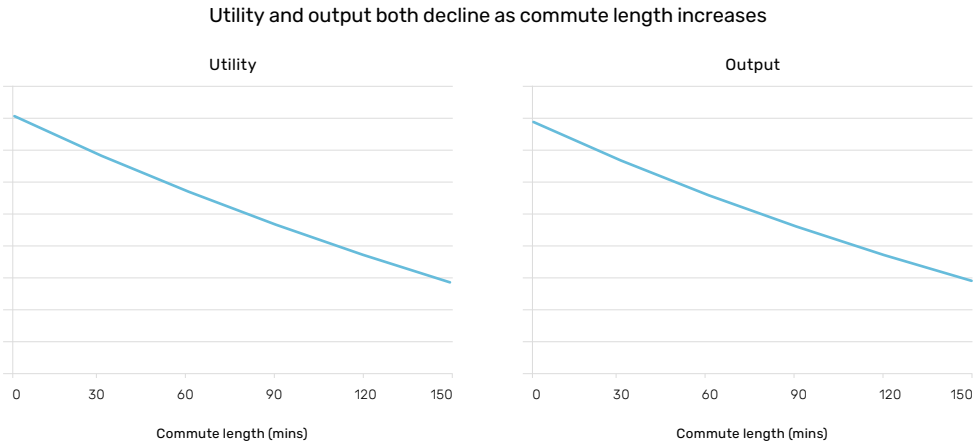
Overall labour supply increases when work from home becomes an option because the commute can be avoided. In this model, the commute does not benefit individuals or firms, except by enabling office based work. It is simply lost time for the individual. This may not be the case for all people – some people may get exercise from riding to work or get pleasure from reading on the train – but it is a reasonable assumption that the opportunity cost of the commute time is higher than any incidental benefit gained from it. Put another way: most people would not commute if they did not have to – they have other things they would rather do.

From the point of view of the individual, this is time that can be spent in paid work or unpaid activities. This division of time between these two activities is determined by the preferences of the individual as set out in the utility function.

The benefit to the firm is not explicitly captured in this model in so far as the firm is a profit maximiser with constant returns to scale technology – profits are zero in equilibrium. Allowing work from home means the individual has more time, some of which they might spend working. This means that firm output can increase through the purchase of additional labour. So, although profit remains zero, the firm expands. In terms of overall welfare, this is a weak Pareto improvement – when some of the commute is avoided, the firm is at least as well off as they were before and the individual is either the same or better off.

Figure 4 shows that as the fixed length of the commute increases, overall utility and output both decrease.

Figure 4. Increases in the commute make everyone worse off



We can see the effect of the commute on the individual's utility from the first order conditions if we consider equation (8), reproduced below.

$$\frac{\alpha_{L_o} L_o^{r-1}}{U^*} - \lambda_H(1+t) + \lambda_C w^o + \lambda_{L_o} = 0$$

$$U^* = \frac{\alpha_{L_o} L_o^{r-1}}{\lambda_H(1+t) - \lambda_C w^o - \lambda_{L_o}}$$

This shows that as the fixed length of the commute increases (the t in the denominator of the second equation), holding all else constant, utility (the left hand side of the equation) will decrease. However, it is worth noting here that we are holding hours worked in the office fixed. It is also plausible that the individual could maintain a similar level of utility by accepting a longer commute and proportionally decreasing their hours worked in the office – such as by moving further away from the city centre and working more from home, a result found by Lennox (2021). As a result, city populations could become more dispersed. Nonetheless, the need to infrequently commute for those with hybrid working arrangements will likely tether many workers to cities – seeing them move at furthest to the outer suburbs rather than relocating to regional areas.

A similar logic implies that higher travel times will, in all likelihood, reduce firm output. Re expressing the equation above to have L_o on the left hand side shows that increased travel times will reduce L_o, holding other variables constant. The corresponding re expression of equation (7) does not contain t at all, except implicitly via the other variables. Substituting the identity for L_o and L_h into the firm's output function, it can be seen that firm output is a negative function, ceteris paribus, of t as well.

The corollary of the above point, is that enabling working from home makes everyone better off as the time saved from avoiding commute can be channelled into work. This effect is larger for people with longer commutes.

Flexible wages produce an efficient allocation of labour

Without wages to determine the level of labour traded in the market, employers and employees must either agree how much home and office work will be done through bargaining, or the firm must set a rule and the worker must decide to take it or leave it.

In the case of bargaining, the outcome will reflect the relative level of bargaining power between employer and employee and the mechanism by which an agreement is reached. This is a complex scenario as it would either require individual bargaining, implying employees would have lower bargaining power on average, or group bargaining, which would make it hard to reflect the diverse preferences of employees.

If the firm sets the rule, then we can show that its optimal ratio of hours are unlikely to be optimal for the individual.

Consider the firm's profit maximisation problem:

$$\pi = p\left(\beta_{L_h} {L_h}^{\rho} + \beta_{L_o} {L_o}^{\rho}\right)^{\frac{1}{\rho}} - w^h L_h - w^o L_o$$

The firm is substituting between the two inputs L_h and L_o. Assume for the moment that the firm believes people are exactly as productive at home as they are in the office and intends to weight the inputs to production equally. This implies that $\beta_{L_h} = \beta_{L_o}$. Because the production function exhibits constant returns to scale, if the wages are forced to be equal then output would only be maximised if $L_h = L_o$.

We can see this if we set $w^h = w^o$ in the firm's first order conditions. Equation (a) becomes:

$$\frac{L_h}{L_o} = \left(\frac{\beta_{L_o}}{\beta_{L_h}}\right)^{\frac{1}{\rho - 1}}$$

Setting the right hand side of the equation to 1 ($\beta_{L_h} = \beta_{L_o}$) will reduce to $L_h = L_o$.

In practice, it is possible that, because home based labour is a new technology, the firm believes work from home is less productive than work in the office (i.e. $\beta_{L_h} < \beta_{L_o}$). In this case the firm will demand more labour hours from the office than from the home. This means the firm and employee get locked into a low work from home scenario.

Once the firm sets the rule, for example $L_h = L_o$, then the individual must supply equal amounts of office and home labour or choose to move to another firm (which in our model means exiting the labour market). Because the individual faces a more complicated maximisation problem this situation will only be utility maximising for the individual under very restrictive conditions.

If we constrain wages to be equal in the case where $L_h = L_o$, then there will be an implied set of parameters (α's, t and r) that are consistent with equal labour in equilibrium. Consider the individual's utility function when we substitute the constraints into it and set wages to be equal:

$$U = \left(\alpha_C(wL_h + wL_o)^r + \alpha_H(\overline{T} - L_h - (1+t)L_o)^r + \alpha_{L_h}{L_h}^r + \alpha_{L_o}{L_o}^r\right)^{\frac{1}{r}}$$

There would be a unique set of parameters that would maximise individual utility. For example, if we compared two individuals with the same substitution parameter who differ in that one has a longer commute, they would need to have a different set of α's to achieve the same level of utility. The individual with the longer commute (i.e. larger t) would have a ratio of α's that are shifted more towards favouring home labour, but the principle would remain the same as with the firm.

This would also be true for any different ratio of L_h and L_o. The firm would determine the ratio $\frac{L_h}{L_o}$ given a particular set of parameters. For each ratio there will be a unique value of α's, t and r that can maximise the objective function of the individual. Considering that (certainly in the short run) the parameters are exogenous to the individual and firm, it is very unlikely that individuals and firms in the economy have the corresponding set of parameters that will maximise their respective objective functions without a price mechanism. This implies that it is unlikely that fixed wages would lead to optimal outcomes.

Without flexible wages, parameters evolve over time

In reality, a differential wage is unlikely to be a practical mechanism for a variety of reasons, such as equity concerns about the people who can only work from home being (potentially) paid less than their office-based counterparts.

Although fixed wages may create a short run mismatch between firms and individuals, it is possible that in practice, the α's and β's may evolve over time for various reasons.

- In instances where work from home is not as productive as work done in the office (i.e. when $\beta_{L_h} < \beta_{L_o}$), firms are likely to invest in order to improve home based productivity.
- Firms that want more labour supplied from the office can also offer non wage inducements to employees to try and increase their relative enjoyment of the office (that is increasing, α_{L_o} relative to α_{L_h}). This could include investing in better office space, lunches and social events.
- Individuals who place a great value on the ability to work from home also have an incentive to increase their marginal product of home based labour to ensure the firm demands more labour from the home. This could be achieved by undergoing training, developing good communication with managers, and minimising distractions at home.

Even with price adjustments, these changes are likely to happen over time as firms and workers experiment with working from home and develop their understanding of what works best for them. The model specified here also does not capture the sorting between heterogenous firms and individuals that we know will happen in the real world to resolve mismatches.

Conclusion

To provide a theoretical foundation for the Productivity Commission's Working from Home research report we developed a simple model of working from home. We constructed a simple hedonic labour model where labour units are broken into labour supplied/demanded from the home and labour supplied/demanded from the office and impose a cost of supplying labour from the office that is borne only by the supplier of labour (i.e. the commute). We assumed that the firm values labour from each location based on its relative productivity, which can vary, while the individual has different preferences for each location.

We found that increased access to work from home increases labour supply. This increase is larger for individuals with longer relative commutes, as the time saved from commuting can be distributed between work and non-work activities. The labour supply increase is largest among those people who have a stronger preference for working from home. We found that the commute is a major cost which is borne entirely by the individual who supples labour and the removal of the constraint which prohibits working from home (i.e. allowing working from home) unambiguously increases individual utility — largely because of this commuting cost. We also found that paying a different wage to office vs home-based labour yields an economically efficient outcome. However, when wages cannot vary by location, firms and workers will likely make adjustments over time to make the distribution of work more efficient; such as by investing in home-based work technologies, or by developing processes to make distributed work more productive.

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