

Education Vouchers and Labour Supply

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Abstract

This paper compares labour supply behaviour under a uniform education voucher system with one involving a means-tested scheme in which the voucher is subject to a taper or withdrawal rate as parental gross income increases. Particular attention is given to the implications of nonlinear budget constraints. Parents maximise a utility function which includes their consumption, leisure and the human capital of children. The human capital production function has inputs consisting of parental human capital and expenditure on education.

JEL Classification: H21; H52; H4

1. Introduction

The large and varied literature on education vouchers has concentrated on the case of universal vouchers. The arguments in favour of education subsidies, whether based on externalities or imperfect capital markets, are generally taken for granted.¹ Studies have concentrated on their effects on geographical mobility, as for example in Nechyba (1999), or mobility between public and private education systems, where the latter is regarded as being of a higher quality; see, for example, Glomm and Ravikumar (1992) and Cardak (1999, 2004). Related analyses include Bearse *et al.* (2000, 2004), Caucutt (2004) and Fernandes and Rogerson (2001). Reviews of many other issues relating to the use and design of voucher systems are contained in West (1997), Cardak and Hone (2003) and Ladd (2002).

Vouchers have widely been judged to be inequality increasing. This observation has lead a number of authors to suggest the use of means-testing; see Bearse *et al.* (2000), Ladd (2002, p.4) Preston (2003, p.84), Cardak (2004, p.22) and González *et al.* (2004). Means-testing is usually advocated on the grounds that it improves 'target efficiency', using the terminology introduced by Beckerman (1979). Despite these suggestions, a rare study of the effects of means-testing of vouchers is by Fernandez and Rogerson (2001), who consider two alternative types of means-testing. First, those below an income threshold are given a fixed voucher, while those above the threshold receive no voucher. Second, those below the threshold receive a voucher

¹ Fernandez and Rogerson (2001) emphasise the market failure rationale for vouchers in terms of imperfect capital markets. Cardak (2004) stressed a fiscal spillover effect whereby vouchers, by increasing growth, increase the tax base.

depending on their income and the amount spent on education. However, their model has fixed labour supplies, and thus exogenous incomes. It is well known that means-testing of social transfers has the effect of introducing kinks in budget constraints, which then involve ranges where the budget set facing each individual is non-convex, as a result of the reduction in the marginal effective tax rate when individuals exhaust their eligibility. This leads to complex labour supply responses.²

In the present context, the introduction of a means-tested voucher involves further complexities in view of the fact that the voucher cannot be sold, but must be devoted to expenditure on the education of children. Furthermore, education expenditure can exceed the value of the voucher. Parents who place a high value on the education of their children may wish to spend more by, for example, sending children to a more expensive school, perceived to be of a higher quality than that obtained from the voucher alone. The incentive to work longer hours to pay for more education for children is clearly affected by the presence of means-testing. And of course there is a significant role for an education production function in producing human capital.

The incentive effects of means-tested education vouchers have not previously been examined. Hence, the aim of this paper is to examine labour supply in the presence of education vouchers combined with a tax and transfer system. It compares a uniform education voucher system with a means-tested scheme in which the voucher is reduced as parental gross income increases. The major feature of means-testing of education vouchers that is examined is the considerable complexity of labour supply arising from the nonlinearity of individuals' budget constraints. Nonlinearities also arise from a restriction that education vouchers can be used only for education investment purposes and cannot be traded among parents. The labour supply implications are considered in the context of a model allowing for the incentive effects on both labour supply and educational investment choices by parents. In view of the emphasis on this neglected feature, the present analysis abstracts from considerations of geographical mobility and the public/private school mix, which have been discussed at length elsewhere.

The framework of analysis is described in the second section. Parents are assumed to maximise a utility function which includes their consumption, leisure and the human capital of children. The latter is generated from a human capital production function with inputs consisting of parental human capital and expenditure on education. An important feature of the model is that parents differ both in their own human capital and their preferences regarding the education of children. The voucher, along with a social dividend and any other non-transfer government expenditure required, is financed from a proportional income tax within a pay-as-you-go system.

Section 3 applies this framework to the case of a uniform, or unconditional, education voucher system. The complications arising from means-testing are examined in section 4. Section 5 examines the form of the government budget constraint, showing the transfer payment as the tax rate is varied, for given parameters of the voucher system. In view of the nonlinearities involved in aggregating over all individuals, numerical methods are used. The evaluation of alternative voucher schemes clearly depends on the criteria and other value judgements adopted. However, section 6 reports results involving the use of a social welfare function reflecting inequality aversion. Brief conclusions are in section 7.

² On means-tested versus universal transfer payments, see Mitchell, Harding and Gruen (1994), Atkinson (1995) and Creedy (1995). However, these studies do not examine education vouchers.

The Framework of Analysis

This section describes the basic model. The utility functions and human capital production function are set out in the first subsection and the tax and transfer system is described in second subsection.

Utility Functions

In the following analysis, in order to avoid problems associated with joint utility maximisation and labour supply decisions, parents are essentially treated as a single individual. The population is made up entirely of such parents, with each person having a single child. This is a standard assumption in all models cited here. The subscript, t , refers to the generation of parents while the subscript, $t + 1$, refers to the generation of children. Each parent has endowments of time, normalised to unity, and human capital, denoted by $h_{i,t}$, for parent i . In the present context human capital actually reflects the productivity, or wage rate, of an individual. Earnings are the only source of income, other than government transfer payments.

Each parent is assumed to derive utility from its own consumption and leisure, C and from the human capital of the child. Let $C_{i,t}$ denote the consumption of the i th parent, where the price is normalised to unity. The leisure of the parent is denoted $L_{i,t}$, and the human capital of the child is denoted $h_{i,t+1}$. The utility function of the parent is assumed, as in Preston (2003), to be Cobb-Douglas, expressed for convenience in logarithmic form:³

$$U_{i,t} = \theta \log C_{i,t} + \alpha \log h_{i,t+1} + (1 - \theta - \alpha) \log L_{i,t} \quad (1)$$

There is no loss of generality in having the coefficients on $C_{i,t}$, $h_{i,t+1}$ and $L_{i,t}$ sum to unity. For convenience, no subscripts have been placed on the parameters here. However, preference heterogeneity plays an important role in the following analysis. The child's human capital results from a production function involving the parent's human capital and expenditure, made entirely by the parent, devoted to the education of the child. Define $E_{i,t}$ as education expenditure, and $h_{i,t}$ as the human capital of the parent. Again, following Preston (2003, p. S76) the human capital production function takes the Cobb-Douglas form:⁴

$$h_{i,t+1} = \delta E_{i,t}^\gamma h_{i,t}^{1-\gamma} \quad (2)$$

The parent's human capital is included on the grounds that much child development occurs inside the home. It is also possible to think of a fixed amount per person of public expenditure being devoted to education, which would influence the size of δ . The

³ In Glomm and Ravikumar (1992) schooling quality enters U rather than human capital of the child; furthermore, the child is assumed to make a decision regarding the choice of work and time spent in education, and the adult has fixed labour supply. Cardak (1999, 2004) uses a similar model to that of Glomm and Ravikumar. In Barse *et al.* (2000) there are no labour supply choices. Fernandez and Rogerson (2001) have a utility function including consumption and the expected value of next period's income of the child, with no labour supply variations.

⁴ Glomm and Ravikumar (1992) had an additional term to allow for the proportion of time spent in education by the child.

function in (2) has the property, in common with all studies discussed here involving education vouchers, that it does not specify a productivity externality. That is, there is no benefit, in the form of a higher wage rate arising from skill complementarities, from the fact that other households invest more heavily in education. As mentioned above, other types of externality, involving social and cultural values, are also used to justify tax-financed subsidies, along with market failure arguments concerning capital markets. The present model contains only a fiscal spillover effect whereby future generations benefit from higher growth and thus a higher tax base. Substituting (2) into (1) gives:

$$U_{i,t} = k + \theta \log C_{i,t} + \alpha \gamma \log E_{i,t} + (1 - \theta - \alpha) \log L_{i,t} \quad (3)$$

where $k = \alpha \{ \log \delta + (1 - \gamma) \log h_{i,t} \}$. This form of the utility function allows the choice of investment, consumption and labour supply by each parent to be determined, given the budget constraint discussed below.

Taxes and Transfers

A non-transferable education voucher, worth $V_{i,t}$, is available to the parent. It is possible to think of the voucher being used to obtain education in the public school system of uniform quality. Parents then have the opportunity of spending more than this and sending children to private schools, where the quality varies. The reasonable restriction is imposed that the voucher can be used only for education purposes – otherwise the voucher would be equivalent to a transfer payment. Each parent may supplement the voucher by spending an additional amount, $S_{i,t}$, on the child's education. Hence expenditure by parent i is given by:

$$E_{i,t} = V_{i,t} + S_{i,t} \quad (4)$$

Clearly, $S_{i,t} \geq 0$, so the restriction, $E_{i,t} \geq V_{i,t}$ must also hold.

The models in Preston (2003), Cardak (1999, 2004), Glomm and Ravikumar (1992), Bearnse *et al.* (2000) and Bearnse *et al.* (2003) have no other forms of transfer payment. This means that all individuals work, since there are no other sources of income. And in means-tested systems, those not eligible for a voucher have fixed labour supplies. However, the addition of transfer payments is more realistic and so the present model has an unconditional transfer payment of b per parent.

Suppose that the voucher and transfer system is financed by a proportional tax on the earnings of parents at the rate, τ . Gross earnings are denoted $Y_{i,t}^G$, so that:

$$Y_{i,t}^G = h_{i,t} (1 - L_{i,t}) \quad (5)$$

Net income, $Y_{i,t}^N$, from earnings and the transfer payment, is given by:

$$Y_{i,t}^N = h_{i,t} (1 - L_{i,t}) (1 - \tau) + b \quad (6)$$

The i th parent's budget constraint is thus:

$$h_{i,t} (1 - L_{i,t}) (1 - \tau) + b + V_{i,t} = C_{i,t} + E_{i,t} \quad (7)$$

In this type of model it is necessary to work in terms of ‘full income’, defined as the income that would be obtained if all time were spent working. Hence the individual is considered to convert the endowment of 1 unit of time into income and then ‘purchase’ desired leisure time, at a price equal to the net wage rate, along with consumption. Full income cannot be fully specified until the precise form of the education voucher is known.

A Uniform Voucher

This section examines parents’ choices when there is an unconditional uniform voucher system. Even with a simple voucher scheme the existence of various thresholds means that budget constraints are nonlinear, thereby complicating labour supply and investment decisions. With a uniform voucher of \bar{V} for each parent:

$$V_{i,t} = \bar{V} \quad (8)$$

The full income of the i th parent, $M_{i,t}$, is therefore given by:

$$M_{i,t} = h_{i,t}(1-\tau) + b + \bar{V} \quad (9)$$

The parent selects $E_{i,t}$, $C_{i,t}$ and $L_{i,t}$ to maximise utility, given by (3) subject to the budget constraint. Since the effective price of leisure per unit is the net wage, $h_{i,t}(1-\tau)$, the budget constraint in (7) above can be rewritten as:

$$C_{i,t} + E_{i,t} + h_{i,t}(1-\tau)L_{i,t} = M_{i,t} \quad (10)$$

There are the additional constraints that $E_{i,t} \geq \bar{V}$ and $L_{i,t} \leq 1$. The most convenient way to consider this optimisation problem is first to examine interior, or tangency, solutions and then to examine whether the inequality constraints are satisfied.

For Cobb-Douglas utility functions, the demand for any good is expressed as the product of two terms. The first is the ratio of the exponent on the good to the sum of all exponents, while the second term is the ratio of full income to the relevant price. The sum of the coefficients on (the logarithms of) C , E and L in (3) is equal to $1 + \alpha(\gamma - 1)$. Hence, for consumption:

$$C_{i,t} = \frac{\theta}{1 + \alpha(\gamma - 1)} M_{i,t} \quad (11)$$

Education expenditure is given by:

$$E_{i,t} = \frac{\alpha\gamma}{1 + \alpha(\gamma - 1)} M_{i,t} \quad (12)$$

And the demand for leisure is equal to:

$$\begin{aligned} L_{i,t} &= \left(\frac{1 - \theta - \alpha}{1 + \alpha(\gamma - 1)} \right) \frac{M_{i,t}}{h_{i,t}(1 - \tau)} \\ &= \left(\frac{1 - \theta - \alpha}{1 + \alpha(\gamma - 1)} \right) \left\{ 1 + \frac{b + \bar{V}}{h_{i,t}(1 - \tau)} \right\} \end{aligned} \quad (13)$$

since $h_{i,t}(1 - \tau)$ is the price of leisure.

These results hold only when the inequality constraints, $E_{i,t} \geq \bar{V}$ and $L_{i,t} \leq 1$, are satisfied. Attention needs to be given to possible corner solutions. From (12), the parent spends more on education than the voucher only when human capital exceeds a threshold, h_E , such that:

$$h_E = \frac{(1-\alpha)\bar{V} - \alpha\gamma b}{(1-\tau)\alpha\gamma} \quad (14)$$

If $h_{i,t} < h_E$, it is necessary to set $E_{i,t} = \bar{V}$. As a consequence of this, consumption and leisure are determined by maximising:

$$U_{i,t} = k' + \theta \log C_{i,t} + (1-\theta-\alpha) \log L_{i,t} \quad (15)$$

where:

$$k' = \alpha \{ \log \delta + (1-\gamma) \log h_{i,t} + \gamma \log \bar{V} \} \quad (16)$$

Applying the standard Cobb-Douglas properties gives consumption and leisure, instead of the above results, as:

$$C_{i,t} = \frac{\theta}{1-\alpha} (h_{i,t}(1-\tau) + b) \quad (17)$$

and:

$$L_{i,t} = \left(\frac{1-\theta-\alpha}{1-\alpha} \right) \left(1 + \frac{b}{h_{i,t}(1-\tau)} \right) \quad (18)$$

Finally, from (18), the parent works if human capital (the wage rate) exceeds a threshold, h_W , where:

$$h_W = \left(\frac{b}{1-\tau} \right) \left(\frac{1-\theta-\alpha}{\theta} \right) \quad (19)$$

For those with $h_{i,t} < h_W$, then $L_{i,t} = 1$, $E_{i,t} = \bar{V}$ and $C_{i,t} = b$. These results show that the relationships between human capital and expenditure on education and consumption take the form of piecewise-linear schedules. Cobb-Douglas utility functions imply that both $E_{i,t}$ and $C_{i,t}$ are linear functions of $h_{i,t}$, between relevant ranges. It is worth stressing that the results here are driven largely by the nonlinear nature of individuals' budget constraints, rather than the unit elasticity of substitution of the Cobb-Douglas form.

Typical forms are shown in figures 1 to 3. Consumption and education expenditure are respectively b and \bar{V} , when $h_{i,t} < h_W$ and the parent does not work. Beyond W consumption jumps to a higher level, associated with the jump in labour supply shown in figure 3. A feature of the model is that there is not a smooth transition into work, as indicated by the horizontal section of the profile in figure 3. This is caused by the fact that the non-transferable education voucher implies that individuals must, over a range of relatively low values of h , effectively spend more of their net income on education than they would otherwise spend. As shown in figure 1, education expenditure does not exceed \bar{V} until $h_{i,t} > h_E > h_W$. The discontinuity in the labour

Figure 1 - Education Expenditure and the Wage Rate

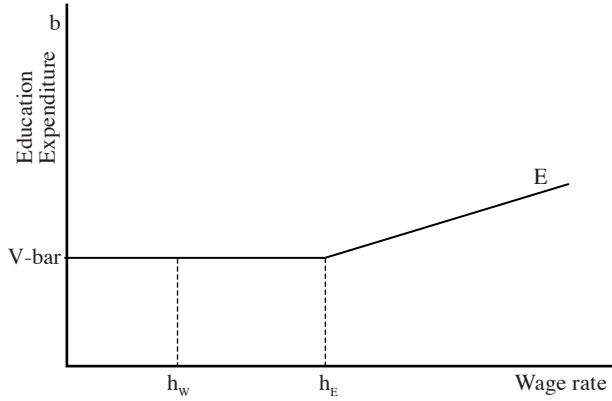


Figure 2 - Consumption and the Wage Rate

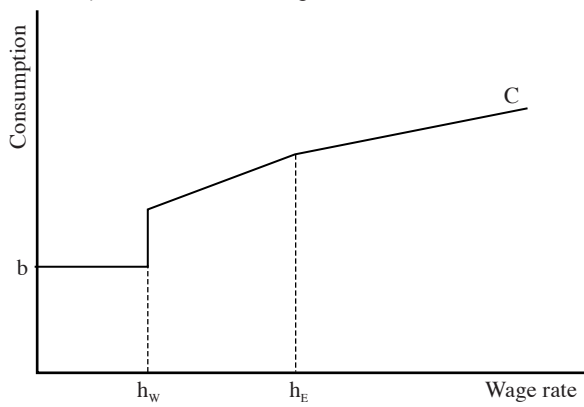
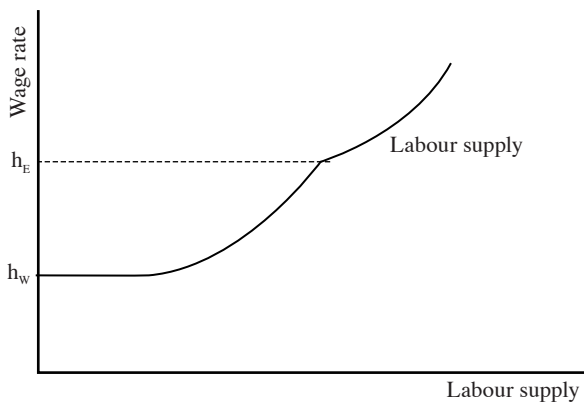


Figure 3 - Labour Supply with Uniform Voucher



supply from non-work to work depends (in part) on the size of the transfer payment relative to the education voucher. A higher value of b means that the threshold value, h_w , is higher, but the initial work hours are lower. Even where $b > \bar{V}$, there is a discrete jump on entry into work, so long as desired education expenditure is lower than \bar{V} . Furthermore, the higher b also means that the threshold h_e is lower, so that education expenditure exceeds the fixed voucher level over a wider range of parent's human capital.

4. A Means-tested Voucher

Suppose the voucher is means-tested and subject to a taper, or benefit withdrawal, rate of β . The maximum voucher, received by those who do not work, is V^* and the voucher received by the i th parent, $V_{i,t}$, is given by:

$$V_{i,t} = \max(V^* - \beta Y_{i,t}^G, 0) \quad (20)$$

As above, $Y_{i,t}^G = h_{i,t}(1 - L_{i,t})$ is the gross earnings of the i th parent. This is a standard form of income testing. Those with $Y_{i,t}^G \geq V^*/\beta$, receive no voucher and must fully fund their child's education from post-tax earnings. However, in setting a value V^* , a minimum level of spending is seen as appropriate by the government. For this reason, a further constraint is imposed on all parents that they must spend at least V^* on education.

The non-convexity in the budget set, introduced by means testing, complicates the labour supply and consumption behaviour of individuals in two ways. First, there can be multiple local optima, with one being at a corner solution involving no labour supply and the other being a tangency position where no voucher is obtained. There can also be simultaneous tangency positions, along the same indifference curve, on both segments of the budget constraint. Secondly there can be discrete jumps in labour supply when a small change in the net wage causes an individual to move between segments of the budget constraint. The critical, or threshold, net wage is the wage giving rise to the two tangency position along a single indifference curve.

The first and second subsections examine consumption and labour supply for those eligible to receive the voucher and those who exhaust their entitlement, respectively. The third subsection considers the possible profiles which can arise from such a system. Some numerical examples are given in the fourth subsection.

Voucher Recipients

For those who are eligible for a means-tested voucher, the budget constraint is:

$$h_{i,t}(1 - L_{i,t})(1 - \tau) + b + V^* - \beta h_{i,t}(1 - L_{i,t}) = C_{i,t} + E_{i,t} \quad (21)$$

which can be rearranged as:

$$\begin{aligned} L_{i,t} h_{i,t}(1 - \tau - \beta) + C_{i,t} + E_{i,t} &= h_{i,t}(1 - \tau - \beta) + b + V^* \\ &= M_{i,t} \end{aligned} \quad (22)$$

Hence for this group, the constraint looks exactly like the universal voucher budget constraint except that the effective income tax rate is $\tau + \beta$ rather than simply τ . Voucher recipients therefore face a higher effective tax rate than those who have

$Y_{i,t}^G \geq V^*/\beta$. This gross earnings threshold translates into a threshold in terms of labour supply, $1 - L_{i,t}$, of $V^*/\beta h_{i,t}$. Hence this range of the budget set can be ruled out immediately if it results in $L_{i,t} < 1 - V^*/\beta h_{i,t}$. As in the case of a uniform voucher, a check must be made to ensure that the resulting value of $E_{i,t}$ is at least V^* . If the constraint does not hold, the appropriate adjustment must be made, following the same procedure as described in the previous section. As before, those who do not work receive the full voucher of V^* , and consume the universal transfer of b . However, if this corner solution applies, it may be only one local optimum. It is necessary to check the possibility that the parent may be better off by working relatively long hours and paying the lower marginal tax rate while receiving no voucher.

Voucher Non-recipients

Those who have exhausted their benefit entitlement face a budget constraint:

$$h_{i,t}(1 - L_{i,t})(1 - \tau) + b = C_{i,t} + E_{i,t} \quad (23)$$

In terms of full income, $M_{i,t}$, this translates to:

$$\begin{aligned} L_{i,t}h_{i,t}(1 - \tau) + C_{i,t} + E_{i,t} &= h_{i,t}(1 - \tau) + b \\ &= M_{i,t} \end{aligned} \quad (24)$$

With this modification, the results for consumption, education expenditure and leisure in equations (11), (12) and (13) apply simply by setting $\bar{V} = 0$. Any tangency solution giving rise to gross earnings, below the threshold above which the voucher is exhausted, must be ruled out as infeasible.

It is necessary to check that the resulting value of $E_{i,t} \geq V^*$, which holds when:

$$h_{i,t} \geq \frac{1 + \alpha(\gamma - 1)}{\alpha\gamma} \left(\frac{V^*}{1 - \tau} \right) - \frac{b}{1 - \tau} \quad (25)$$

If individuals are constrained to set $E_{i,t} = V^*$, an adjustment must be made to the values of leisure and consumption, again following the approach discussed in the previous section. It is possible that this adjustment rules out this range of hours worked where no voucher is received, that is if the resulting $L_{i,t}$ is greater than $1 - V^*/\beta h_{i,t}$.

Possible Profiles

A possible relationship between the gross wage rate (human capital) and consumption is shown in figure 4. This is similar to that shown in figure 2 except that at wage rate, h_s , a small increase causes the individual to jump from working and receiving a means-tested voucher to a higher labour supply where no voucher is received. Figure 4 shows a situation in which the individual is voluntarily spending more on education than the maximum voucher, while continuing to receive a reduced (means-tested) voucher.

Figure 4 - Consumption with Means-tested Voucher

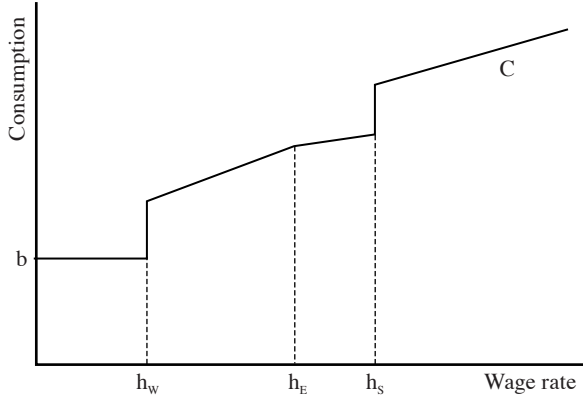


Figure 5 - Consumption with Means-tested Voucher: A Binding Education Expenditure Constraint

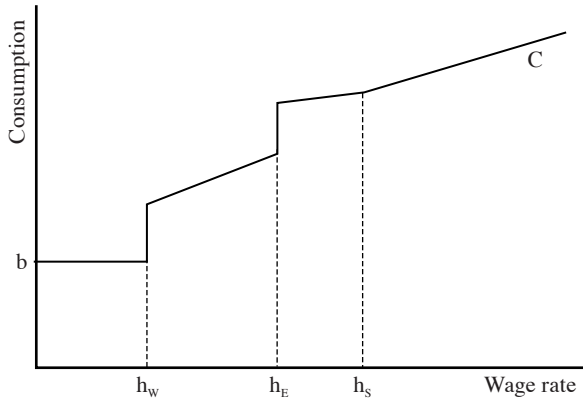
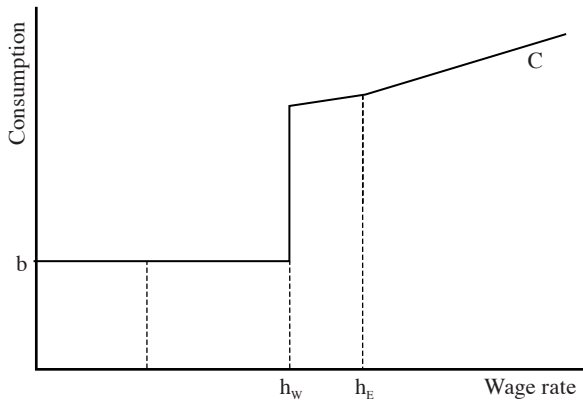


Figure 6 - Consumption with Means-tested Voucher: A High Taper Rate and Binding Education Constraint



The consumption profile could take alternative forms. Two possibilities are shown in figures 5 and 6. In figure 5, all those receiving a means-tested voucher are constrained by the need to spend more on education than they would otherwise wish, and some of those who exhaust their entitlement to the voucher face a similar binding constraint. Figure 6 illustrates the situation in which the taper rate is so high that it is never optimal to work and receive a voucher, but some of the workers face the binding constraint whereby $E_{i,t}$ is set equal to V^* . Yet another possibility, not illustrated here, $h_E < h_W$ is that in figure 6.

Individual Profiles

It is of interest to consider numerical examples. Individual labour supply profiles are shown in figures 7 to 10. In each case $\theta = 0.45$, $\alpha = 0.35$ and $\gamma = 0.6$. The only difference between figures 7 and 8 is that in the former the taper rate β is equal to 0.1 while in the latter it is increased to 0.3. The figures clearly show the extent to which the lower taper implies that a positive voucher is received over a wider range of the wage rate. Furthermore, education expenditure exceeds the minimum required (determined by the maximum voucher V^*) over a slightly wider range, and parents work over a wider range of h . For the lower taper in figure 7, the relevant wage thresholds are $h_W = 2300$, $h_E = 3800$ and $h_S = 27800$. For the higher taper in figure 9, the relevant wage thresholds are $h_W = 3300$, $h_E = 5300$ and $h_S = 10800$.

Figure 9 illustrates a labour supply profile for higher levels of the transfer and the maximum voucher and of the tax rate. In this case, all individuals below $h_W = 7000$ do not work, and human capital must reach $h_E = h_S = 17000$ until education expenditure exceeds the minimum required amount: this arises after the discrete jump takes place onto the range where no voucher is received. Finally, figure 10 has a relatively high transfer payment, financed by a higher income tax rate, but there is a uniform voucher ($\beta = \theta$). Here education expenditure exceeds the universal voucher of 2800 when $h_{i,t} > h_E = 5700$.

Figure 7 - Labour Supply and Expenditure Profiles: $V^* = 2200$; $b = 3500$;
 $\beta = 0.1$; $\tau = 0.2$

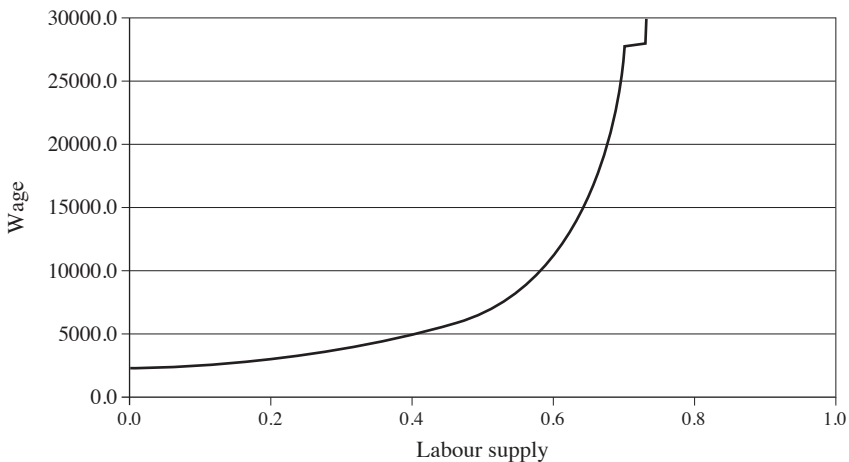


Figure 8 - Labour Supply and Expenditure Profiles: $V^* = 2000$; $b = 3500$;
 $\beta = 0.3$; $\tau = 0.2$

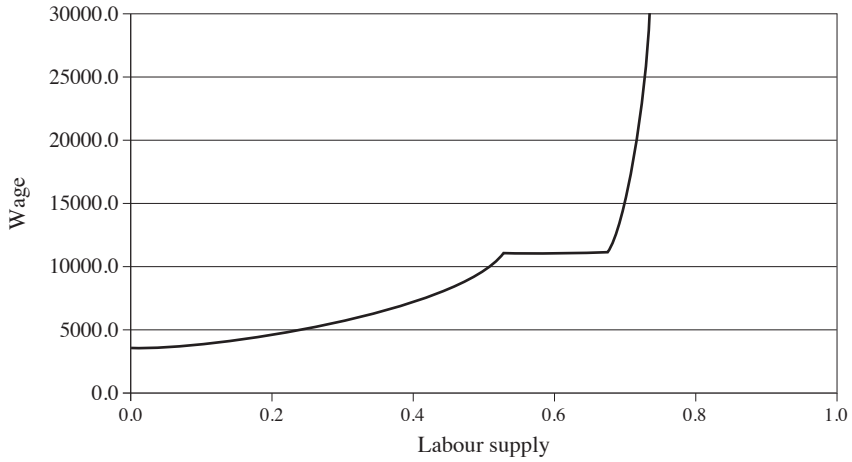


Figure 9 - Labour Supply and Expenditure Profiles: $V^* = 3000$; $b = 4000$;
 $\beta = 0.3$; $\tau = 0.4$

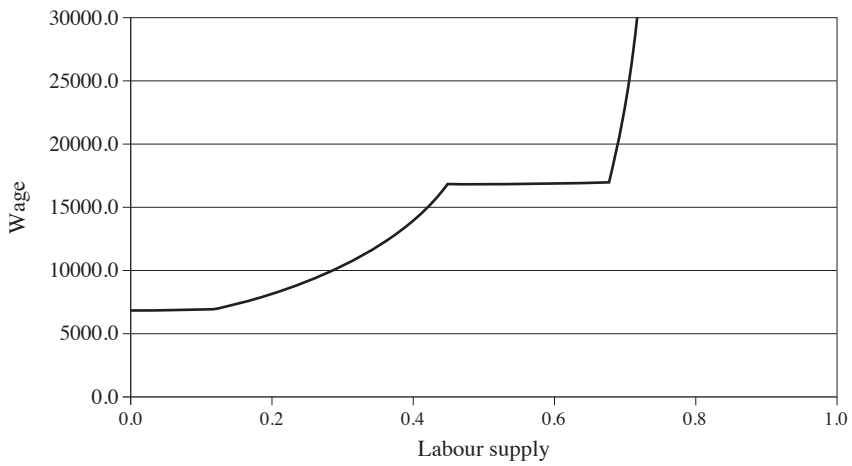
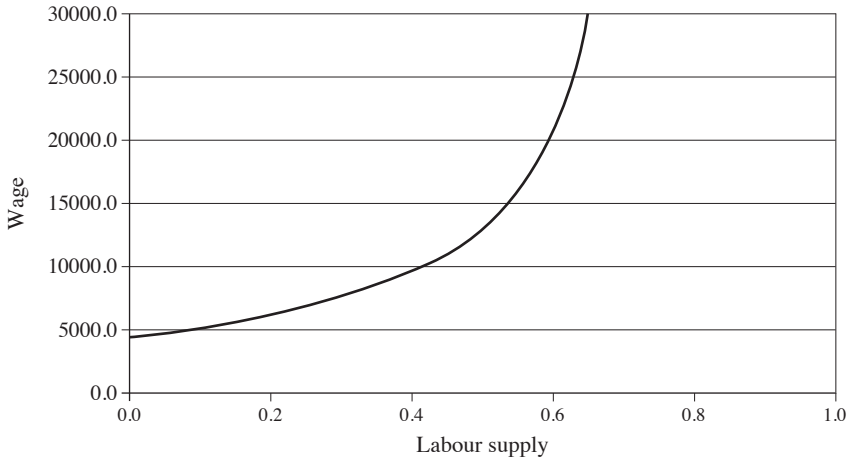


Figure 10 - Labour Supply and Expenditure Profiles: $V^* = 2800$; $b = 5500$;
 $\beta = 0$; $\tau = 0.45$



5. The Government Budget Constraint

Suppose that transfers and vouchers are financed on a pay-as-you-go basis for each generation of parents. With n parents, total income tax revenue is equal to $\tau \sum_i^n Y_{i,t}^G$. The total cost of the voucher system is $\sum_i^n V_{i,t}$ and the total cost of transfer payments is nb . Hence the government's budget constraint is:

$$\tau \sum_i^n Y_{i,t}^G = \sum_i^n V_{i,t} + nb \quad (26)$$

and:

$$\tau \bar{Y}^G = \bar{V} + b \quad (27)$$

where \bar{Y}^G and \bar{V} are average earnings and average voucher received by parents, and the time subscripts are omitted for convenience. The simple property that $\tau = (\bar{V} + b) / \bar{Y}^G$, so that the tax rate is equal to the ratio of benefits per person to average earnings, is nevertheless deceptive. In fact, it is highly nonlinear in view of the dependence of earnings on the tax and transfer parameters. The present section therefore examines the budget constraint using a simulated population of individuals.

Suppose parents' preferences regarding the human capital of children, determined by the parameter α of the utility function, are correlated with human capital, $h_{i,t}$. Heterogeneous preferences regarding education expenditure (where these varied independently of human capital) in majority voting education models were introduced by Cardak (1999), who found that they increased inequality in the private education model. Preston (2003) assumed a uniform distribution of α . This correlation can be modelled by specifying a joint distribution of h and α . Dropping subscripts for convenience, suppose that h and α are jointly lognormally distributed as:

$$\Lambda(h, \alpha \mid \mu_h, \mu_\alpha, \sigma_h^2, \sigma_\alpha^2, \rho) \tag{28}$$

Assume that individuals have a common value of θ , the coefficient on consumption in the utility function. Preferences for leisure also vary, since the coefficient is obtained as $1 - \theta - \alpha$. A simulated population may be obtained as follows. First, select a random observation from the marginal distribution of h . If v_i is a random variable drawn from an $N(0,1)$ distribution, the corresponding value of the i th parent's human capital, h_i , is obtained as $h_i = \exp(\mu_h + v_i \sigma_h)$. A corresponding value of α_i is obtained from the conditional distribution of α , given h . Let u_i denote another random draw from an $N(0,1)$ distribution. Then:

$$\alpha_i = \exp \left\{ \mu_\alpha + \rho \frac{\sigma_\alpha}{\sigma_h} (\log h_i - \mu_h) + u_i \sigma_\alpha (1 - \rho^2)^{1/2} \right\} \tag{29}$$

In the following examples, $\mu_h = 10$ and $\sigma_h^2 = 0.5$. In addition, a minimum value of h_i of 2000 is imposed. A simulated population of 5000 parents is used. In the majority of cases presented, it is assumed that $\sigma_\alpha = 0.2$, and $\rho = 0.5$, so that there is a tendency for those parents with relatively high human capital to have a higher preference for increasing that of their children; on ρ see Patacchini and Zenou (2007).⁵

The consumption, education expenditure and leisure choices of parents are independent of the parameter δ , so for convenience this is set at 2.6 in all cases presented below. This coefficient is chosen to ensure that the average value of human capital grows from one generation to the next. In addition, given the form of the constant, k , in the utility function, it affects the absolute value of utility. This constant term differs among parents given the variability in α .

For each value of τ , the government budget constraint is solved iteratively to produce the corresponding value of the unconditional transfer payment, b . A trial value of b , say b_0 , is used to examine each parent's choices, and hence the corresponding values of \bar{Y}^G and \bar{V} are computed. Then the resulting transfer, b_1 , is obtained using:

$$b_1 = \tau \bar{Y}^G - \bar{V} \tag{30}$$

If $b_1 > b_0$, the trial value is increased slightly, or if $b_1 < b_0$ the trial value is reduced, and another iteration is carried out until convergence is reached. It would not of course be appropriate to try to solve the budget constraint for τ , given a value of the transfer payment, b , because it is possible to have two different tax rates corresponding to any transfer level: holding other parameters constant, the feasible value of b first increases and then falls as τ is increased. However, the restriction on the range of τ , determined by the assumed value of β , means that b does not fall in all cases; examples are shown below.

An indication of the effect of varying the income tax rate, for given values of the other parameters, is given in figures 11 and 12, which are based on simulated populations of 5000 parents. In each case $\theta = 0.45$, $\bar{\alpha} = 0.35$, $\sigma_\alpha^2 = 0.02$ and $\gamma = 0.6$. These values are chosen for illustrative purposes only. Extensive sensitivity analyses found that the basic results are unaffected by parameter values. The difference in the

⁵ It is most convenient to specify the arithmetic mean value, $\bar{\alpha}$, and then to determine the appropriate value μ_α of using $\mu_\alpha = \log \bar{\alpha} - \frac{1}{2} \sigma_\alpha^2$.

two diagrams is the maximum voucher level, set at 2000 and 5000 in figures 11 and 12 respectively, and results are shown in each case for two taper rates.

For the lower value of $V^* = 2000$, the higher taper rate allows a slightly higher transfer payment to be financed, for a given value of τ , though the range of income tax rates is substantially restricted with the higher taper rate. The resulting social welfare is higher with the lower taper, for a given income tax rate. For the case where $V^* = 5000$, the profiles in figure 12 are slightly different. The minimum value of τ must be higher with the lower taper rate, because the greater generosity of the voucher system means that insufficient revenue is obtained unless the tax rate is sufficiently high. The profile of b is somewhat different in this case, as it has a kink at a tax rate of around 0.34. The transfer payment can actually be higher for a high taper rate compared with a low taper.

Figure 11 - Variation in Transfer Payment with Tax Rate: $V^* = 2000$

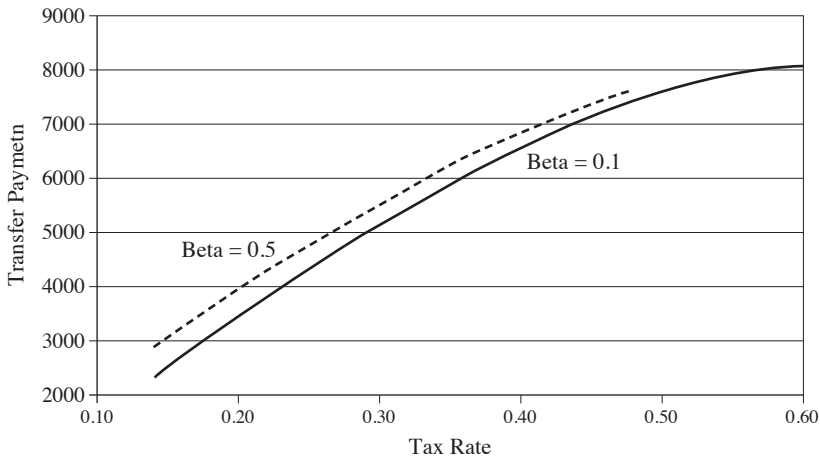
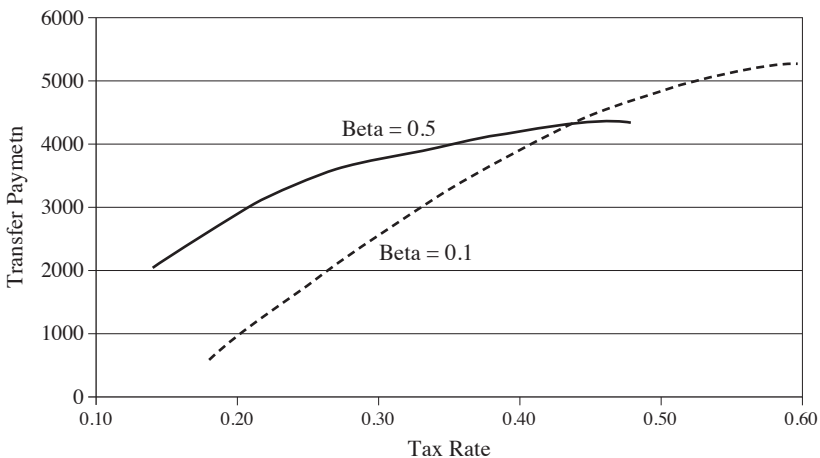


Figure 12 - Variation in Transfer Payment with Tax Rate: $V^* = 5000$



These results are explained by the behaviour of labour supply in the two ranges of the income tax rate. For example, with preferences of $\theta = 0.45$ and $\alpha = 0.35$, and tax parameters $V^* = 5000$, $b = 3500$, $\tau = 0.3$ and $\beta = 0.5$, the three wage (or human capital) thresholds are $h_w = 12,8000$, $h_s = 17,400$ and $h_e = 24,300$. Hence as the wage increases, individuals begin to work and receive a means tested benefit, then jump to the range of the budget constraint where no benefit is received, while continuing to spend only the minimum amount (of 5,000) on education, until a wage of 24,300 is reached. However, with the same parameters except that $b = 3800$ and $\tau = 0.38$, individuals jump directly from not working (and receiving the full voucher) to the segment of the budget constraint where no voucher is received; that is, $h_w = h_s = 18,700$, and furthermore, $h_e = 26,900$. The simulations have a distribution of the parameter α , as explained above, but over the higher income tax rates the small number of people receiving a reduced voucher allows the unconditional transfer to be higher with a high taper than with the low taper rate.

6. Social Welfare Functions

Previous sections of this paper have concentrated on disentangling the labour supply effects of voucher schemes. The broader evaluation of alternative voucher systems is highly problematic in view of the varying objectives behind them and the dynamic elements involved. A wide range of criteria could indeed be applied. This section takes an admittedly narrow perspective. It follows that used in optimal income tax modelling, where alternatives are evaluated using a social welfare function reflecting the value judgements of an independent (omniscient) judge. As in the standard approach used in the optimal tax literature, the social welfare function is assumed to take the individualistic Paretean form:

$$W = \sum_{i=1}^n \frac{U_i^{1-\varepsilon}}{1-\varepsilon} \quad (31)$$

where ε is the constant relative inequality aversion parameter of the judge and only a single generation of parents is considered.

Using a simulated population of parents, as described in section 5, it is possible to search numerically for values of the policy variables which maximise the welfare function. There are two tax rates, τ and β , and the values of V^* and b (if $\beta = 0$, then $\bar{V} = V^*$), but one degree of freedom is lost because of the government budget constraint in (27). For each combination of voucher parameters examined, β and V^* , a range of tax rates τ is considered, where the restriction $\beta + \tau < 1$ must limit the upper value of the tax rate. In addition, a minimum value of τ applies, given the need to finance the voucher system. At each iteration the optimal consumption, labour supply and education expenditure decisions of each of the 5000 simulated parents are computed. For each inequality aversion parameter, ε , W is evaluated. The combination of values giving the highest W (for each ε) then gives the 'optimal' set of tax parameters.

The major result is that for all preference and human capital production function parameters examined, the value of β which maximises ε was found to be zero; that is, the optimal voucher system using this narrow social welfare criterion is one involving an unconditional voucher.

The sensitivity of results is shown in table 1. Case B, in the first row of the table, refers to the 'base' case used, and variations in the parameters are shown in the following rows. The other parameters are $\mu_h = 10.0$, $\sigma_h^2 = 0.5$, $\rho = 0.5$ and $\sigma_\alpha = 0.02$. It was also found that there is little effect of increasing the value of σ_α . Under the columns headed 'Full Model', the optimal transfer payment is higher than the voucher level, $V^* = \bar{V}$. The optimal tax rate and transfer levels show little sensitivity to the degree of inequality aversion of the social welfare function. Furthermore, a higher aversion involves a higher optimal social dividend, with little effect on the voucher. Similarly, the parameter γ of the human capital production function has little effect on the optimal tax rate, but does lead to a higher optimal voucher.

Table 1 - Optimal Policy Variables for Alternative Parameter Values: In all Cases Optimal $\beta = 0$

Case	θ	$\bar{\alpha}$	γ	ε	τ	$V^* = \bar{V}$	b
B	0.45	0.35	0.6	0.5	0.47	2800	5674
1	0.45	0.35	0.6	0.1	0.46	2800	5550
2	0.45	0.35	0.6	1.5	0.48	2700	5895
3	0.45	0.35	0.6	3.0	0.50	2800	6026
4	0.45	0.35	0.4	0.5	0.46	1900	6100
5	0.45	0.35	0.5	0.5	0.46	2300	5886
6	0.45	0.35	0.7	0.5	0.47	3500	5134
7	0.4	0.35	0.8	0.5	0.47	3600	5178
8	0.45	0.30	0.6	0.5	0.44	2100	5196
9	0.40	0.35	0.6	0.5	0.45	2500	4812
10	0.40	0.30	0.6	0.5	0.43	2000	4392
11	0.35	0.45	0.6	0.5	0.46	3800	4373

Notes: $\mu_h = 10.0$; $\sigma_h^2 = 0.5$; $\rho = 0.5$; $\sigma_\alpha = 0.02$.

The redistributive element involved in the voucher is reduced by the assumed positive correlation between parents' human capital and their preference for childrens' human capital. For the 'base case' a correlation coefficient of $\rho = -0.5$ produces a slightly higher optimal voucher of $V^* = \bar{V} = 3200$, with b reduced to 5096 and $\tau = 0.46$ (and as in other cases, $\beta = 0$). The negative correlation means that, on average, α is higher for those with lower h , and given the positive skewness in the distribution of h , it is not surprising that moving from a positive to a negative correlation is similar to that of increasing $\bar{\alpha}$.

In obtaining the W-maximising values, no constraints were placed on the range of possible values. If there is a constraint on τ and hence gross revenue, the case for targeting the voucher is stronger. Indeed, the desire to reduce total expenditure usually lies behind arguments for means-testing (though this is rather arbitrary, since gross revenue is reduced if the system is administered by paying only net transfers and collecting only net tax). Table 2 reports, again for the 'base case', optimal values of the other policy variables if τ is constrained to be below the corresponding value shown in table 1. The restriction has little effect on the optimal value of V^* , with the burden falling on the reduction in b . The restriction means that the optimal voucher system

now involves means testing. However, even with a very low value of τ , the taper rate is low. Indeed, restricting V^* to be above the unconstrained socially optimal value, and constraining the income tax rate, such that $V^* = 5500$ and $\tau = 0.35$, produces an optimal β of only 0.12. Here β is effectively the only policy instrument over which there is freedom to choose, since b is determined to satisfy the government budget constraint.

Table 2 - Optimal Values with Restricted Income Tax Rate

<i>Restricted τ</i>	β	V^*	b
0.15	0.14	3600	1675
0.20	0.13	3600	2478
0.25	0.12	3700	3140
0.30	0.10	3600	3808
0.35	0.08	3300	4593

7. Conclusions

This paper has compared potential labour supply behaviour in two simplified models of a uniform education voucher system and a means-tested scheme in which the voucher is subject to a taper or withdrawal rate as parental gross income increases. Parents maximise a utility function which includes their consumption, leisure and the human capital of children. A human capital production function has inputs consisting of parental human capital and expenditure on education. In the means-tested case, expenditure on education by each parent is constrained to be at least as large as the maximum voucher available.

In examining labour supply, along with the choice of consumption and education expenditure, it was necessary to pay close attention to the complexities introduced by inequality constraints and non-convexities in the budget set of parents. The government faces a budget constraint such that the voucher and a social dividend are financed from a proportional income tax, within a pay-as-you-go system involving the generation of parents.

While recognising the existence of a wide range of evaluation criteria which may be used, alternative schemes were evaluated using a standard social welfare function defined in terms of the utility of parents, and including an aversion to inequality. To find the parameter combination maximising the welfare function, a systematic search was carried out over three of the policy variables, the income tax rate, the maximum voucher and the voucher taper rate, with the transfer payment being determined by the government's budget constraint (which was solved iteratively in view of the nonlinearity involved). For all combinations, a uniform voucher was found to be optimal using this criterion, unless a binding constraint was placed on the maximum tax rate. It would be of interest in future work to examine the implications for growth and inequality in a multi-generational framework.

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