The Role of Home Production in Voting Over Taxes and Expenditure

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Abstract

This paper investigates, first, how allowance for subsistence activities, or home production, affects the standard results in models involving the majority choice of the tax rate in a flat tax – basic income scheme. The paper extends the analysis of home production to choices regarding the composition of government expenditure, in situations where there is a tax-financed pure public good in addition to a transfer payment, conditional on a given tax rate. The effect of home production is to reduce the transfer payment in each model, but the effect is small.

1. Introduction

There is now a substantial literature on the effects of home production on labour supply, welfare and growth, with much emphasis being given to the role of joint decision making in households. However, the present paper considers the potential influence of home production in a rather different context. Its aim is to investigate how the inclusion of subsistence activities, or home production, may influence voting outcomes regarding the choice of taxes and the composition of government expenditure. In order to compare results with standard models in this tradition, following earlier work of, for example, Roberts (1977) and Meltzer and Richard (1978), a simple specification is used in which single individuals make labour supply and consumption choices.

Two related questions are examined. First, majority voting over the tax rate in the familiar linear tax and transfer system is considered.[1] With a proportional tax and a universal benefit, the government budget constraint implies that voting is over only one dimension, the tax rate. In this case it may, for example, be expected that there is a larger 'tax base effect' of an increase in the tax rate, compared with the standard model which excludes home production. The non-participation option no

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[1] Attention is restricted here to majority voting rather than other democratic decision mechanisms. An alternative approach involves a stochastic voting framework; see Persson and Tabellini (2000). This leads to maximisation of a function which resembles a social welfare function, and closed-form solutions are generally not available. Tridimas and Winer (2005) considered probabilistic voting, with home production, using quasi-linear utility functions and concentrating on the choice of public goods and a linear income tax.

longer implies that consumption is restricted to the transfer payment. The fact that individuals can substitute home production for goods purchased in markets, as well as substituting leisure for work when the tax rate rises, may result in a preference for relatively lower tax rates, and thus lower transfer payments.

Second, this paper examines the potential role of home production in democratic choices regarding the composition of government expenditure, in situations where there is a tax-financed pure public good in addition to a transfer payment, conditional on a given tax rate. Hence in this framework, as so often in practice, there is a separation between taxing and expenditure decisions. After pointing out that this is a common assumption, Tridimas (2001, p. 308) suggests that this, 'is less restrictive than it first appears, since in practice governments are often constrained in the policy instruments that they may vary at anyone time'. Bearse *et al*. (2001), who examine majority voting over a uniform transfer and public education, also assume that the tax rate is given exogenously. Again the role of the government budget constraint means that voting is unidimensional: on difficulties raised by multidimensional voting, see Mueller (2003, pp. 87-92).[2]

It has long been established that, in the first case mentioned above – but without home production – a majority voting equilibrium exists in which the median voter's preferred tax rate is a function of the ratio of the median wage rate to the arithmetic mean wage. An increase in the skewness of the wage rate distribution (that is a reduction in the ratio) is associated with a higher equilibrium tax rate and therefore a more redistributive tax-transfer system.[3] A standard diagram is used with the basic income, or transfer payment, on the vertical axis and the proportional tax rate on the horizontal axis; see Figure 1 below.[4] Individuals with lower wage rates have flatter upward sloping indifference curves, while non-workers have horizontal indifference curves since, not paying tax, they prefer only the highest transfer possible. Each voter's preferred position involves a tangency between the highest indifference curve and the concave government budget constraint. Hence the lower the median relative to the mean, the higher is the majority choice of tax rate.[5]

However, empirical evidence regarding such a relationship, involving cross-country data, has been mixed. For a review of evidence, see Borck (2007) and Harms and Zink (2003), and on problems raised in testing this type of model empirically, see Lind (2005). In the context of time series evidence for particular countries, the variation in inequality is typically too small to establish an effect.

A similar property arises in models of the democratic choice of expenditure composition combining public goods and redistributive transfers. A positive relationship

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[2] In some models involving taxation and more than one form of government expenditure, a two-stage process is envisaged in which voting over the tax rate takes place, where voters have information about the conditional choice of government expenditure.

[3] It can be shown that the majority choice satisfies the condition, $1-(y_m / \bar{y}) = |\eta_{\bar{y}\cdot\tau}|$, where y_m and $\bar{y}$ are respectively the median and arithmetic mean gross income, and $|\eta_{\bar{y}\cdot\tau}|$ is the absolute elasticity of average income with respect to the tax rate, τ. The left hand side of this expression can be interpreted as a measure of income inequality.

[4] It is discussed by, for example, Mueller (1989, pp. 512-514).

[5] Mueller (1989), as is usual, draws indifference curves as convex. However, it is shown below that they may be slightly concave.

can be established between inequality and the proportion of expenditure devoted to the inequality-reducing transfer payment: on this relationship, see Creedy and Moslehi (2009). Again cross-country empirical evidence is equivocal.

This raises the question of whether comparisons among countries, particularly involving both developed and developing countries, where home production or subsistence activities might be thought to vary substantially, are significantly affected. The question concerns the possible extent to which differences in taxes and expenditure composition, among democratic countries, may be explained by different degrees of importance attached to subsistence activities, compared with other factors such as cultural differences which are also known to vary substantially. The main aim of the present paper is thus to consider this question. In pursuing this problem, it is also necessary to consider the precise form of specification of home production such that the models are reasonably tractable.

Following a description of the framework of analysis in section 2, section 3 considers voting over the tax rate in a model in which there are two goods, one of which is produced at home, in addition to leisure. The sensitivity of the choice of tax rate to the degree of importance attached to home production is examined. 'Importance' here is affected both by the preference for the home produced good in the utility function and the productivity of time spent on home production. Section 4 considers the case where voting concerns the division of government expenditure between transfer payments and a pure public good, conditional on a given tax rate. Again the extent to which the relationship between the majority choice of expenditure share and the ratio of the median to the arithmetic mean wage is influenced by variations in the importance of home produced goods is considered. Section 5 extends the analysis to allow individuals' wage rates to be affected by public good expenditure. Brief conclusions are in section 6.

2. The Framework of Analysis

This section extends the widely used partial equilibrium model of a 'pure' tax and transfer system, where individuals have different abilities which are reflected in their wage rates.[6] A fixed endowment of time is divided among leisure, work and home production. To obtain some idea of potential orders of magnitude, it is useful to obtain explicit solutions for the majority choice of the tax rate using a specific form for the utility function.[7] The present paper uses the Cobb-Douglas form, but considers alternative specifications. The first stage is to obtain the indirect utility function, expressed in terms of the tax parameters, along with the government's budget constraint.

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[6] As a partial equilibrium model, only the supply side of the labour market is examined, so that labour supply variations have no effect on the wage rate distribution. It is referred to as a 'pure tax and transfer' system because no consideration is given to non-transfer expenditure. In the present context this can easily be added to the government budget constraint following the usual approach of assuming that such additional expenditure does not enter individuals' utilities or their wage rates.

[7] Quasi-linear preferences are particularly simple for text-book examples, but are not used here. Hindricks and Myles (2006, pp. 503-5) discuss majority voting where utility is consumption less (half) the square of labour supply, and show that the median voter's preferred tax rate is $(1 - y_m / \overline{y})$ $\{2 - y_m / \overline{y}\}$, where y_m and $\overline{y}$ are median and arithmetic mean income respectively. Persson and Tabellini (2000, chapter 6) also give an example using quasi-linear preferences.

Suppose that individual i buys an amount, x_i, of a marketed good at price, p, and produces y_i of a home produced good using h_i units of time, according to:

$$y_i = \theta h_i^\delta \tag{1}$$

Other inputs into home production, arising from endowments of the individual, are subsumed into the term, θ. These endowments may include, for example, a fixed holding of land and capital goods in the form of tools. It is not required to assume that this is the same for all individuals. If the production function were to involve inputs of amounts of the market-purchased good, x, the model would become significantly more complex.[8]

The individual consumes l_i units of leisure and the total endowment of time is 1, so that the time devoted to paid work is $1 - l_i - h_i$. Using the Cobb-Douglas form, the utility function can be written:

$$U = x_i^\alpha y_i^\phi l_i^\gamma \tag{2}$$

so that, after substituting for y_i:

$$U = x_i^\alpha \left(\theta h_i^\delta\right)^\phi l_i^\gamma \tag{3}$$

Writing $\beta = \theta\phi$ and ignoring the constant θ^ϕ, this can be rewritten as:

$$U = x_i^\alpha y_i^\beta l_i^\gamma \tag{4}$$

It is convenient below to write $\alpha + \beta + \gamma = \rho$. The standard model, which excludes home production, is thus obtained by setting $\beta = 0$. Utility therefore takes the basic Cobb-Douglas form in terms of the consumption of a market-purchased good and the time devoted, separately, to leisure and home production. The latter does not generate utility directly but does so via the production function in (1).

If a constant elasticity of substitution (CES) utility function were used instead of (2), this would not, when combined with (1), give rise to an equivalent CES in terms of hours of home production, as does (4). Furthermore, the CES does not give rise to a linear relationship between earnings and the wage rate, so the government budget constraint is considerably more complex than the present case. An alternative way of looking at home production would be to suppose that instead of having two goods, one of which can be produced at home, there is just one good which may either be produced at home or purchased at price p. From the point of view of consumption, they are otherwise the same. Home and market amounts consumed are x_s and x_p respectively. Utility is thus $U = \left(x_p + x_s\right)^\alpha l^{1-\alpha}$, where $x_s = \theta h_s^\delta$. However, a problem with this formulation is that it becomes intractable.

With a tax and transfer system involving a proportional tax applied to all earnings at the rate, τ, and a basic income of b, the budget constraint, where w_i is the wage rate, is:

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[8] Greenwood *et al.* (1995) allow for the purchase of inputs, in a real business cycle model.

$$px_i + w_i(1-\tau)(h_i+l_i) = w_i(1-\tau) + b = M_i \tag{5}$$

where M_i is 'full income'.[9] Using the standard properties of the Cobb-Douglas utility function, involving fixed expenditure proportions, the individual's optimum values are given by:

$$x_i = \frac{\alpha}{\rho}\frac{M_i}{p} \tag{6}$$

$$h_i = \frac{\beta}{\rho}\frac{M_i}{w_i(1-\tau)} \tag{7}$$

$$l_i = \frac{\gamma}{\rho}\frac{M_i}{w_i(1-\tau)} \tag{8}$$

Thus, as expected, high wage individuals devote relatively more time to working in the labour market, rather than taking leisure or engaging in home production. Where the opportunity cost of time is lower, it is better to spend more time in home production. Gross earnings, $y_i = w(1-h_i-l_i)$, are:

$$y_i = \left(\frac{\alpha}{\rho}\right)w_i - \left(\frac{\rho-\alpha}{\rho}\right)\left(\frac{b}{1-\tau}\right) \tag{9}$$

This expression takes the same form as the case where there is no home production: the only difference concerns the value of the coefficients on the wage rate and basic income. This applies only if w_i exceeds a minimum wage, w_{min}, required to induce positive labour supply.

In order to obtain the government's budget constraint, aggregation must be carried out over all individuals. If $\bar{y}$ is arithmetic mean earnings, then from (9):

$$\bar{y} = \frac{1}{N}\sum_{w>w_{min}}\left\{\left(\frac{\alpha}{\rho}\right)w_i - \left(\frac{\rho-\alpha}{\rho}\right)\left(\frac{b}{1-\tau}\right)\right\} \tag{10}$$

and letting $F_1(w_{min})$ and $F(w_{min})$ denote respectively the proportion of total wage (rates) and the proportion of people with $w < w_{min}$:[10]

$$\bar{y} = \bar{w}\frac{\alpha}{\rho}\left\{1 - F_1(w_{min})\right\} - \left(\frac{\rho-\alpha}{\rho}\right)\left(\frac{b}{1-\tau}\right)\left\{1 - F(w_{min})\right\} \tag{11}$$

where $\bar{w}$ is the arithmetic mean wage rate. This expression is nonlinear in view of the fact that w_{min} depends on b and τ, so it is not possible to express b as a convenient function of τ. However, the analysis is tractable if it is assumed that relatively few

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[9] This is equivalent to a situation in which the individual sells all the endowment of labour time at the going wage and 'buys back' the time required for leisure and home production at a price equal to the net wage.

[10] These correspond to the ordinate and abscissa of the Lorenz curve of wage rates at the point where $w = w_{min}$.

individuals do not work, so that F_1 and F are small and can be neglected: this produces a linear relationship between arithmetic means of y and w. This assumption in fact has a negligible effect on the shape of the government's budget constraint in the relevant region (that is, for tax parameters around those preferred by the median voter), though it may have a small effect on the absolute size of the basic income.

The government's budget constraint in this 'pure' transfer scheme is simply $b = \tau \bar{y}$ so that substituting for $\bar{y}$ and rearranging gives:

$$b = \frac{\alpha \bar{w} \tau (1-\tau)}{\rho - \alpha \tau} \tag{12}$$

Substituting optimal values of consumption, along with (12), into full income, $M_i = w_i(1 - \tau) + b$, gives indirect utility, V_i, in terms of the tax rate, τ, as:

$$V_i = k\left\{w_i(1-\tau)\right\}\left(1 + \left(\frac{\bar{w}}{w_i}\right)\frac{\frac{\alpha}{\rho}\tau}{1-\frac{\alpha}{\rho}\tau}\right)^\rho \tag{13}$$

Where $k = \alpha^\alpha \beta^\beta \gamma^\gamma / (\rho^\rho p^p)$ depends on the price of goods in the market and the parameters of the utility function.

3. The Majority Choice of Tax Rate

Early approaches to examining the democratic choice of the size of government had to consider the existence of a majority voting equilibrium when preferences are double peaked.[11] Double-peaked preferences exist for some individuals because, after the point where they move to the non-participation corner solution, they prefer to see the tax rate increase until total revenue (and hence the transfer payment) reaches a maximum. Roberts (1977) showed that a voting equilibrium exists if there is 'hierarchical adherence' (or 'agent monotonicity'), such that the ordering of individuals by income is independent of the tax rate. It is easy to show that the present model satisfies hierarchical adherence, so the median voter theorem can be invoked with the median voter being identified as the person with the median wage rate. The voting equilibrium is obtained in subsection 3.1. The effects on the choice of tax rate of variations in preferences for, and the efficiency of, home production are examined in subsection 3.2.

The Median Voter's Choice

Denoting the median wage by w_m, the majority choice of tax rate, τ_m, is the solution to $dV_m/d\tau = 0$. Differentiation of (13) and rearrangement gives τ_m as the appropriate root of the following quadratic:

$$\tau^2\left(\frac{\alpha}{\rho}\right)^2\left(1-\frac{w_m}{\bar{w}}\right) - \tau\left(1+\frac{\alpha}{\rho} - 2\frac{\alpha}{\rho}\frac{w_m}{\bar{w}}\right) + \left(1-\frac{w_m}{\bar{w}}\right) = 0 \tag{14}$$

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[11] Extensions within the Roberts-Meltzer-Richard framework include, for example, Galasso (2003) who considers fairness and redistribution.

The two roots of this quadratic equation are examined in Appendix A where it is shown that the largest root can be ruled out as it is greater than one.

The wage rate distribution is, as with all income distributions, positively skewed, so that $\bar{w} > w_m$. As mentioned in the introduction, a general result in the literature on majority voting is that a reduction in $w_m/\bar{w}$, that is a movement of the median voter further below the arithmetic mean, is associated with an increase in the median voter's desired tax rate and thus transfer payment, making the system more redistributive. Redistribution is across the arithmetic mean, since the effective average tax rate is negative for $y_i < \bar{y}$ and positive for $y_i > \bar{y}$. Hence, as the median wage tends to the arithmetic mean wage, the majority choice of tax rate tends to $\tau_m = 0$.[12] Thus it is often said that more basic inequality leads to voting for a more redistributive tax structure.

This result continues to hold where home production exists. Using the expression for the appropriate root of (14), given in Appendix A, it is possible to show that the derivative of the tax rate, τ_m, with respect to $w_m/\bar{w}$ is negative; that is, $\partial\tau_m/\partial(w_m/\bar{w}) < 0$. Thus, as in the basic model, reducing wage rate inequality reduces the majority choice of tax rate.

Variations in Beta

The question of interest here is how the existence of home production affects the choice of tax rate. As home production enters the majority choice of tax rate through the coefficient, β, the exponent on time spent in home production in the utility function, this question concerns the model's comparative static properties with respect to β. This could in principle be examined by differentiating the appropriate root of (14) with respect to β. Also, the slope, $\partial\tau_m/\partial(w_m/\bar{w})$, could be differentiated with respect to β, in each case bearing in mind that $\rho = \alpha + \beta + \gamma$. However, this approach does not yield unequivocal results, so that a more indirect route is needed.

Figure 1 - The Median Voter's Choice of Tax Rate

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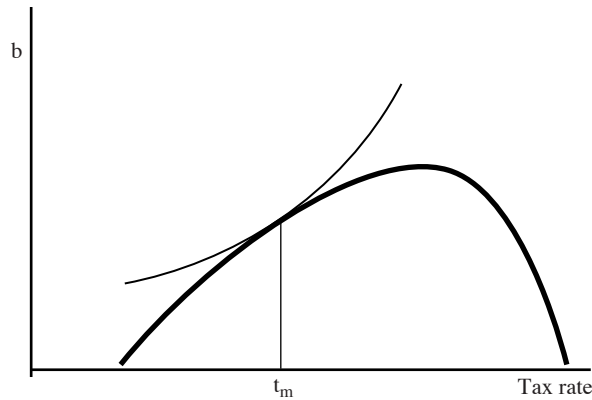
[12] In general, the ratio $w_m/\bar{w}$ is not directly a measure of inequality (since it is equal to 1 for distributions which are symmetric around $\bar{w}$), but in the case of positively skewed distributions it can be taken to reflect inequality as well as the skewness of the wage rate distribution. For example, if w is lognormally distributed as $\Lambda(w|\mu,\sigma^2)$ where μ and σ^2 are respectively the mean and variance of logarithms, it can be shown that $w_m/\bar{w}$ depends only on σ^2.

Further insight can be obtained by considering individuals' preferences in (b, τ) space. The majority voting equilibrium, illustrated in figure 1, is characterised by tangency between the median voter's highest indifference curve and the government budget constraint. With b and τ on vertical and horizontal axes respectively, any change leading indifference curves of the median voter to become steeper, and the government budget constraint (over the relevant – that is upward sloping – range) to become flatter, has the effect of unambiguously reducing the choice of tax rate. It is shown in Appendix B that, although both the budget constraint and indifference curves become flatter, the net effect is that an increase in β reduces τ_m. This is illustrated in figure 2, which shows the variation in τ_m with $(w_m/\overline{w})$ for a range of values of β. In producing the figure, the value of α is set to 0.7 and it is convenient to set $\gamma = 1 - \alpha$ (so that $\rho = 1 + \beta$). The introduction of home production, or an increase in β, not only reduces the value of τ_m but also involves a very slight reduction in the extent to which it varies with $w_m/\overline{w}$. An increase in β can arise from either an increase in preferences for the home produced good, ϕ, or an increase in the productivity of time spent in home production, δ. In each case there is a stronger incentive to devote more time to home production, involving a greater opportunity cost of working. The median voter thus wishes to compensate by having a slightly lower income tax rate.

Figure 2 - Median Voter's Preferred Tax Rate and Ratio $w_m/\overline{w}$
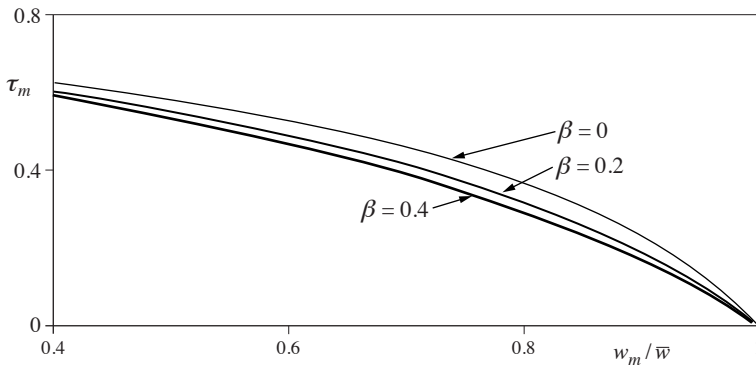
It is also of interest to consider the way in which time allocation varies as β increases. An increase in β, the coefficient on home production time in utility, is expected to involve a shift away from leisure. It is shown here that it also leads to a small reduction in labour supply. First, the partial effects on leisure, ℓ, and time in home production, h, of an increase in β can be seen by differentiating the above expressions for optimal choices, giving:

$$\eta_{l,\beta} = \frac{\beta}{l} \frac{\partial l}{\partial \beta} = -\frac{\beta}{\rho} \tag{15}$$

and

$$\eta_{h,\beta} = \frac{\beta}{h}\frac{\partial h}{\partial \beta} = 1 - \frac{\beta}{\rho} \tag{16}$$

Hence $\eta_{l,\beta} + \eta_{h,\beta} = 1$. An increase in β therefore leads to a shift from leisure towards home production, but the two changes are not equal. There is a small effect on labour supply, since:

$$\frac{\partial(1-l-h)}{\partial \beta} = -\left(\frac{\partial l}{\partial \beta} + \frac{\partial h}{\partial \beta}\right) \tag{17}$$

$$= -\frac{\alpha M_i}{\rho^2 w_i(1-\tau)} < 0$$

Hence, the partial effect of an increase in β is to reduce labour supply for all wage groups. However, the increase in β has been seen above to lead to a reduction in the majority choice of τ and a reduction in the value of b, since the government budget constraint becomes flatter. The latter reduction has the effect of increasing labour supply. Hence the change in labour supply resulting from both changes depends on the individual's wage rate.

These results concern the term β, but this term itself depends both on the productivity of time devoted to home production, as reflected in the coefficient, δ, as well as the relative weight, ϕ, attached to consumption of the home-produced good in the utility function. But since $\beta = \delta\phi$, both these terms enter in a symmetric fashion and their effects cannot be distinguished.

4. Voting on the Composition of Expenditure

This section extends the model of section 2 by introducing a pure public good which is tax financed. It examines the median voter's preferred allocation of tax revenue between transfer payments and the public good.[13] In concentrating on the composition of expenditure, the tax rate is considered to be exogenously determined, as mentioned in the introduction. This means that there is again only one degree of freedom in choosing the transfer and public good expenditure and voting concerns just one dimension.

Consider the model in section 2 which has two goods, one of which is produced at home. Suppose that, in addition, there is a tax-financed amount of a pure public good, Q_G, where the cost of production per unit is constant and equal to p_G, (the price of the private marketed good is p, as above). The augmented utility function is thus:[14]

$$U_i = x_i^\alpha h_i^\beta l_i^\gamma Q_G^\eta \tag{18}$$

The budget constraint facing each individual is the same as in (5). The utility maximising amounts, x_i, h_i and l_i are exactly the same as in equations (6) to (8) in section 2. Similarly, individual i's earnings are the same as given in (9).

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[13] This section therefore extends the results of Creedy and Moslehi (2009), whose model does not include home production.
[14] A feature of the Cobb-Douglas form in this context is that it avoids a situation where expenditure per person tends to zero as population size increases.

However, the form the government's budget constraint must be modified to allow for the need to raise extra revenue to finance expenditure of $G = p_G Q_G$ on the public good. The government budget constraint becomes:

$$b = \tau \bar{y} - G/N \tag{19}$$

where N is the number of individuals. Hence (12) is easily modified by the inclusion of the term in G/N, so that:

$$b = \frac{\frac{\alpha}{\rho}\tau\bar{w} - G/N}{1 + \left(\frac{\rho-\alpha}{\rho}\right)\left(\frac{\tau}{1-\tau}\right)} \tag{20}$$

where again $\rho = \alpha + \beta + \gamma$. The problem here is to obtain the preferred expenditure levels of G and b for a given tax rate. The indirect utility function, modified by the addition of the public good and substituting the transfer payment from the government budget constraint (20), can be written in term of the policy variable, Q_G, as:

$$V_i = \frac{k\left(\frac{1-\tau}{\rho-\alpha\tau}\right)^{\rho}}{\left\{w_i(1-\tau)\right\}^{\rho-\alpha}} \left\{\rho\, w_i + \alpha\tau(\bar{w} - w_i) - \frac{p_G Q_G}{N}\right\}^{\rho} Q_G^{\eta} \tag{21}$$

It can be shown that $d^2 V_i/dQ_G^2 < 0$ if $\alpha + \beta + \gamma < 1$. Hence preferences are singled-peaked and the majority choice of expenditure on public goods is obtained from $dV_m/dQ_G = 0$. This gives, after some manipulation:

$$\frac{G_m}{N} = \frac{p_G Q_{G,m}}{N} = \bar{w}\left(\frac{\eta}{\rho+\eta}\right)\left\{\frac{w_m}{\bar{w}} + \frac{\alpha}{\rho}\tau\left(1 - \frac{w_m}{\bar{w}}\right)\right\} \tag{22}$$

Hence the expenditure per capita on the public good, as a proportion of $\bar{w}$, depends on the preference parameters, the tax rate, and the ratio $w_m/\bar{w}$. It increases linearly with τ and $w_m/\bar{w}$. The resulting value of b_m is given by appropriate substitution of G_m/N into (20):

$$b_m = \bar{w}\frac{\left(\frac{1}{\rho+\eta}\right)\left\{\alpha\tau - \eta\frac{w_m}{\bar{w}}\left(1 - \frac{\alpha}{\rho}\tau\right)\right\}}{1 + \left(\frac{\rho-\alpha}{\rho}\right)\left(\frac{\tau}{1-\tau}\right)} \tag{23}$$

and $b_m/\bar{w}$ is also a linear function of $w_m/\bar{w}$ but a nonlinear function of the exogenous tax rate, τ. An important implication of the Cobb-Douglas preferences is that this ratio does not depend on the cost of the public good per unit relative to the price of the marketed private good. Combining (23) and (22) shows that the majority choice of the ratio of the transfer payment to public good expenditure per capita, R_m, depends on the given tax rate, the preference parameters and, importantly, the ratio, $w_m/\bar{w}$. Further analysis shows that $dR_m/d(w_m/\bar{w}) < 0$, so that increasing equality is associated with a lower R_m and hence a reduced emphasis on a redistributive expenditure share.
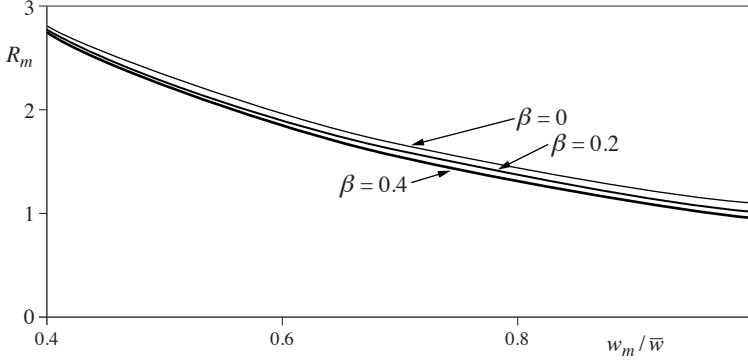
Figure 3 - Expenditure Share and Wage Ratio

Figure 3 shows the relationship between R_m and $w_m/\overline{w}$, again for $\alpha = 0.7$ and $\gamma = 1 - \alpha$, for three different values of β. It can be seen that home production, as modelled here, has little effect on this relationship. Just as it involved a slightly lower tax rate, and hence transfer payment, when considering voting over the tax rate, it implies a slightly lower ratio of expenditure on transfers relative to the public good.

5. An Endogenous Wage Rate Distribution

This section considers the situation in which the public good expenditure affects the wage rate distribution directly.[15] This may be modelled by supposing that each individual's wage depends on G, which may be thought of in terms of basic education. Suppose, for simplicity, that:

$$w_i = w_{0i}^\theta G^{1-\theta} \tag{24}$$

where w_{0i} is person i's 'basic' skill level. This specification implies that G involves an equal proportional increase in all individuals' wage rates and thus does not affect their inequality. However, a higher value of θ produces more inequality.[16]

As in the previous section, income taxation, at the rate τ, finances an unconditional transfer of b and the public good expenditure of G. Each individual consumes goods, x_i, priced at p per unit, along with leisure l_i, and devotes h_i to home production. Again, the majority voting outcome is obtained by first deriving the indirect utility function. In the present context, the productivity enhancing public good is not considered to generate utility directly.[17] Thus, each individual maximises:

$$U_i = x_i^\alpha h_i^\beta l_i^\gamma \tag{25}$$

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[15] The context is still a partial equilibrium model, where interactions with the labour demand side of the economy are ignored.

[16] The variance of log-wage rates is θ^2 multiplied by the variance of log w_0.

[17] Adding public good consumption benefits in the utility function could be added here, as in the previous section. However, having two types of public good, one of which is wage enhancing, produces considerable problems associated with multidimensional voting.

Here it is convenient to impose $1 = \alpha + \beta + \gamma$. The budget constraint is:

$$px_i + w_i(1-\tau)(h_i + l_i) = w_i(1-\tau) + b = M_i \qquad (26)$$

Hence, from the standard Cobb-Douglas properties, $x_i = \alpha M_i/p$, $h_i = \beta M_i/(w_i(1-\tau))$ and $l_i = \gamma M_i/(w_i(1-\tau))$. Earnings $y_i = w_i(1 - h_i - l_i)$ are:

$$y_i = \alpha w_i - (1-\alpha)\frac{b}{1-\tau} \qquad (27)$$

and (if most people work), $\bar{y}$, arithmetic mean earnings, are obtained from (27) with $\bar{w}$, the arithmetic mean value of wage rates, instead of w_i. Indirect utility is therefore:

$$V_i = \left(\frac{\alpha M_i}{p}\right)^\alpha \left(\frac{\beta M_i}{w_i(1-\tau)}\right)^\beta \left(\frac{\gamma M_i}{w_i(1-\tau)}\right)^\gamma \qquad (28)$$

which can be rewritten, letting $K = (\alpha/\rho)^\alpha \beta^\beta \gamma^\gamma$, as:

$$V = K(w(1-\tau))^\alpha \left\{1 + \frac{b}{w(1-\tau)}\right\} \qquad (29)$$

The government's budget constraint is $b + G = \tau\bar{y}$, so that the basic income in terms of G is:

$$b = \frac{\alpha\tau\bar{w} - G}{1 + (1-\alpha)\tau/(1-\tau)} \qquad (30)$$

Aggregating over (24) gives the arithmetic mean wage rate as:

$$\bar{w} = \bar{w}_\theta^\theta G^{1-\theta} \qquad (31)$$

where $\bar{w}_\theta = \left(\frac{1}{N}\sum w_{0i}^\theta\right)^{1/\theta}$. Finally, substitute for $\bar{w}$ in (30) and then for b into (29) to get indirect utility in terms of G:

$$V_i = K\left(w_{0i}^\theta G^{1-\theta}(1-\tau)\right)^\alpha \left\{1 + \frac{\alpha\tau\bar{w}_\theta^\theta - G^\theta}{w_{0i}^\theta(1-\tau)\{1 + (1-\alpha)\tau/(1-\tau)\}}\right\} \qquad (32)$$

Setting $dV_i/dG = 0$, for the median voter, gives the choice of G, G_m, as:

$$G_m = \left[\frac{w_{0m}^\theta(1-\tau)\{1 + (1-\alpha)\tau/(1-\tau)\} + \alpha\tau\bar{w}_\theta^\theta}{1 + \frac{\theta}{\alpha(1-\theta)}}\right]^{\frac{1}{\theta}} \qquad (33)$$

From (30):

$$R_m = \frac{b_m}{G_m} = \frac{\alpha\tau\bar{w}_\theta^\theta G_m^{-\theta} - 1}{1 + \frac{(1-\alpha)\tau}{1-\tau}} \qquad (34)$$

and using (33) gives, after rearrangement:

$$R_m = \frac{\frac{\theta\tau}{(1-\theta)(1-\alpha\tau)} - \left(\frac{w_{0m}}{\overline{w}_\theta}\right)^\theta}{\alpha\tau + (1-\alpha\tau)\left(\frac{w_{0m}}{\overline{w}_\theta}\right)^\theta}(1-\tau) \tag{35}$$

Direct comparisons with the previous section cannot of course be made because here the government expenditure does not enter individuals' utility functions. However, the median voter's preferred expenditure ratio b_m/G_m is again a function of the ratio of the median voter's wage rate to the arithmetic mean wage rate, since $\left(\frac{w_{0m}}{\overline{w}_\theta}\right)^\theta = \frac{w_m}{\overline{w}}$. The question of interest is whether home production has a potentially large effect on this relationship, that is whether it is shifted substantially by variations in β.

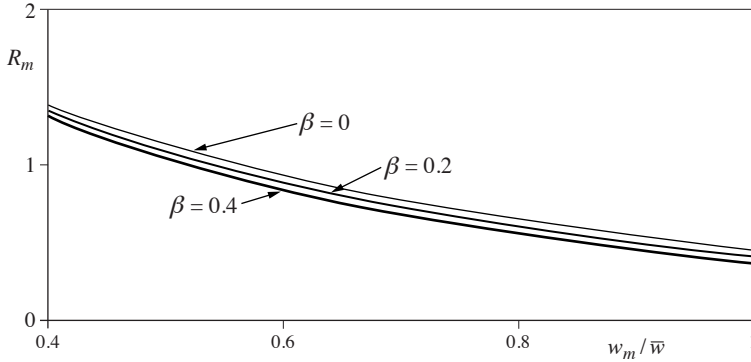Figure 4 - Expenditure Share and Wage Ratio with Endogenous Wages

Figure 4 shows the relationship between R_m and $w_m/\overline{w}$, for three different values of β. Bearing in mind that $\alpha = 1 - \beta - \gamma$. In this figure $\alpha = \gamma = (1-\beta)/2$; however, it was found that changing α and γ give similar results. It can be seen that home production has little effect on the ratio of expenditure on transfer payments to public good expenditure. Furthermore, the downward sloping profile of R_m is relatively flat, as with the model of the previous section.

6. Conclusions

This paper has examined the implications of allowing for home production in modelling three types of democratic choice. First, majority voting over tax and benefit levels was examined in a pure transfer system with endogenous labour supply. Second, the choice of the share of transfer payments in total expenditure was considered in a model in which the tax rate is exogenously fixed but there is also a tax-financed pure public good. Third, the division between transfers and a public good was considered where the latter has an effect on the wage rate distribution, but does not enter individuals' utility functions directly.

The specification of home production implies that a Cobb-Douglas utility function in terms of amounts consumed of a marketed good and a home produced

good (along with leisure) can be re-expressed as a function of the time devoted to home production. The analysis was simplified by the assumption that the minimum wage in the population is sufficient to ensure that most individuals work, producing a convenient form of government budget constraint which allows explicit solutions to be obtained. Both the tax rate in the first model and the expenditure share in the second model were found to depend on the ratio of the median voter's wage to the arithmetic wage. This general property has of course been established earlier for models which make no allowance for home production.

The introduction of home production in these models was found to have little effect on the democratic choice of tax and transfer levels and on the choice of expenditure composition. Attempts to examine empirically the relationship between either the level of transfers or the expenditure share and the ratio of median to average wages have produced mixed results, using cross-sectional data for a range of democratic countries. It is likely that a range of other factors are relevant in the determination of transfers and expenditure shares. The present paper has shown that, even where the extent of home production may be expected to vary significantly, its exclusion from empirical models is not likely to bias results significantly. This negative result is in fact convenient for empirical work, given the difficulty of obtaining information regarding the time spent in home production.

Appendix A
Majority Voting and Two Roots of the Quadratic

In order to find the majority choice of tax rate the two roots of the quadratic equation (14) need to be examined. Writing this quadratic as $A\tau^2 + B\tau + C = 0$, the roots are given by the standard expression $\left\{ -B \pm \sqrt{B^2 - 4AC} \right\}/2A$. The term $B^2 - 4AC$ is given by:

$$B^2 - 4AC = \left(1 + \frac{\alpha}{\rho} - 2\frac{\alpha}{\rho}\frac{w_m}{\overline{w}}\right)^2 - 4\left(\frac{\alpha}{\rho}\right)^2 \left(1 - \frac{w_m}{\overline{w}}\right)^2 \tag{A.1}$$

which, after rearranging, becomes:

$$B^2 - 4AC = \left(1 - \frac{\alpha}{\rho}\right)\left(1 + 3\frac{\alpha}{\rho} - 4\frac{w_m}{\overline{w}}\frac{\alpha}{\rho}\right) \tag{A.2}$$

So that the two roots are:

$$\tau_m = \frac{1 + \frac{\alpha}{\rho} - 2\frac{\alpha}{\rho}\frac{w_m}{\overline{w}} \pm \sqrt{\left(1 - \frac{\alpha}{\rho}\right)\left(1 + 3\frac{\alpha}{\rho} - 4\frac{w_m}{\overline{w}}\frac{\alpha}{\rho}\right)}}{2\left(\frac{\alpha}{\rho}\right)^2\left(1 - \frac{w_m}{\overline{w}}\right)} \tag{A.3}$$

It can be shown that the largest root is greater than unity since:

$$1 + \frac{\alpha}{\rho} - 2\frac{\alpha}{\rho}\frac{w_m}{\overline{w}} + \sqrt{\left(1 - \frac{\alpha}{\rho}\right)\left(1 + 3\frac{\alpha}{\rho} - 4\frac{w_m}{\overline{w}}\frac{\alpha}{\rho}\right)} \tag{A.4}$$

$$> 2\left(\frac{\alpha}{\rho}\right)^2\left(1 - \frac{w_m}{\overline{w}}\right)$$

After much manipulation it can be shown that this condition reduces to:

$$0 > \left(1 - \frac{w_m}{\overline{w}}\right)\left(\frac{\alpha}{\rho} - 1\right)\left\{\frac{\alpha}{\rho} + \frac{w_m}{\overline{w}}\left(1 - \frac{\alpha}{\rho}\right)\right\} \tag{A.5}$$

Of the three terms in parentheses, only the middle term is negative. Hence this condition always holds. Therefore only the lowest root needs to be considered.

Appendix B
Variations in β and the Choice of Tax Rate

The indirect utility function for workers in terms of b and τ can be written as:

$$V_i = k\{w_i(1-\tau)\}^\alpha \left(1 + \frac{b}{w_i(1-\tau)}\right)^\rho \tag{B.1}$$

The slope of an indifference curve is:

$$\left.\frac{db}{d\tau}\right|_{V_i} = -\frac{\partial V_i/\partial \tau}{\partial V_i/\partial b} \tag{B.2}$$

$$= \frac{w_i}{\rho}\left(\alpha + \frac{(\alpha-\rho)b}{w_i(1-\tau)}\right)$$

The sign of the first derivative is undetermined, but it can be shown that over the relevant range of taxes it is increasing. On the other hand the negative sign of the second derivative, $-(\rho-\alpha)b/\rho(1-\tau)^2$, shows that indifference curves are slightly concave in (b,τ) space.[18] This property – which also holds in the basic model where there is no home production – does not seem to have been recognised in the literature, where convex indifference curves are usually drawn.

From (B.2), the effect of a change in on the slope of indifference curves is:

$$\frac{d}{d\beta}\left(\left.\frac{db}{d\tau}\right|_{V_i}\right) = -\frac{\alpha w_i}{\rho^2}\left(1 + \frac{b}{w_i(1-\tau)}\right) < 0 \tag{B.3}$$

and for a given τ the indifference curves get flatter. A change in β also causes the government budget constraint, $b = \tau\overline{y}$, to change. The slope of this is:

$$\left.\frac{db}{d\tau}\right|_R = \overline{y} + \tau\frac{d\overline{y}}{d\tau} \tag{B.4}$$

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[18] Alternatively, writing the equation of the indifferenc curve as $b = (\overline{V}_i/k)^{\frac{1}{\rho}}\{w_i(1-\tau)\}^{\frac{\rho-\alpha}{\rho}} - w_i(1-\tau)$, the first derivative is $\left.\frac{db}{d\tau}\right|_{V_i} = -(\overline{V}_i/k)^{\frac{1}{\rho}}((\rho-\alpha)/\rho)w_i^{\frac{\rho-\alpha}{\rho}}(1-\tau)^{\frac{-\alpha}{\rho}} + w_i$. The sign of this is generally undetermined. However, it applies only for the range of τ for which labour supply is positive, and is therefore positive. For τ beyond the point where the individual does not work, the indifference curves become horizontal. The second derivative is $\left.\frac{d^2b}{d\tau^2}\right|_{V_i} = -(\alpha/\rho)(\overline{V}_i/k)^{\frac{1}{\rho}}((\rho-\alpha)/\rho)w_i^{\frac{\rho-\alpha}{\rho}}(1-\tau)^{\frac{-\alpha}{\rho}-1} < 0$. This is negative, implying that indifference curve are actually slightly concave in (b,τ) space.

and the effect of a change in β on this slope is:

$$\frac{d}{d\beta}\left(\frac{db}{d\tau}\bigg|_R\right) = \frac{d\bar{y}}{d\beta} + \tau\,\frac{d}{d\beta}\left(\frac{d\bar{y}}{d\tau}\right) \tag{B.5}$$

From the expression for y above:

$$\frac{d\bar{y}}{d\tau} = -\left(\frac{\rho-\alpha}{\rho}\right)\frac{b}{(1-\tau)^2} \tag{B.6}$$

and:

$$\frac{d}{d\beta}\left(\frac{d\bar{y}}{d\tau}\right) = -\frac{\alpha}{\rho^2}\frac{b}{(1-\tau)^3} \tag{B.7}$$

Furthermore:

$$\frac{d\bar{y}}{d\beta} = -\frac{\alpha}{\rho^2}\left(\bar{w}+\frac{b}{1-\tau}\right) \tag{B.8}$$

Hence:

$$\frac{d}{d\beta}\left(\frac{db}{d\tau}\bigg|_R\right) = -\frac{\alpha}{\rho^2}\left(\bar{w}+\frac{b}{(1-\tau)^2}\right) < 0 \tag{B.9}$$

Hence the government budget constraint also becomes flatter. This means that there are opposing tendencies on the preferred value of τ. The flattening of the indifference curves leads towards an increase in τ while the flattening of the budget constraint leads towards a reduction in τ. Thus the question, in determining whether the change in β leads to a reduction in the median voter's choice of τ, is whether the change (in absolute terms) in the slope of the budget constraint is greater than that of the indifference curve, at the initial τ. Since $w_m < \bar{w}$ and $0 < 1-\tau < 1$, it can be seen that:

$$\left|\frac{d}{d\beta}\left(\frac{db}{d\tau}\bigg|_R\right)\right| > \left|\frac{d}{d\beta}\left(\frac{db}{d\tau}\bigg|_{V_i}\right)\right| \tag{B.10}$$

An increase in β therefore reduces τ_m.

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